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ABSTRACT

For the case of several interacting chiral superfields the propagators for the unconstrained superfield potentials in the 'shifted' theory, where the supersymmetry is explicitly broken, are derived in a compact form. They are used to compute one-loop effective potential in the general case, while a superfield calculation of renormalized effective potential to two loops for the Wess-Zumino model is performed.

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I. INTRODUCTION

For the globally supersymmetric theories [1] the superfield [2] formulation is very economical to calculate quantum corrections. The cancellation of (higher) divergences associated with the bosonic loops with those of the fermionic loops is automatically taken care of through the superpropagators. The number of supergraphs required to be considered is greatly reduced compared to that needed in the component formulation. The non-renormalization theorems may be shown directly. The superfield path integral formulation is a powerful calculational tool, for example through the background field method [3], the possibility of performing a change of variables to derive Ward identities apart from compactness. The superfield formulation has now been developed sufficiently [3),4] so as to allow manageable calculations to higher order loops.

We derive in Sec.II for the case of several interacting chiral superfields the superpropagators of the unconstrained superpotentials in the presence of a classical background [5),6]. In Sec.III we discuss the superfield tadpole [5] and bubble methods for calculating the effective potential using the shifted theory propagators. The expressions for the one-loop effective potentials are derived and a procedure for the two-loop case as well as for renormalization is indicated. In Sec.IV we discuss in detail the superfield calculation of effective potential upto two loops for the case of a single chiral superfield. The computation is performed in a modified minimal subtraction scheme as well as in a scheme where the renormalization constants are functions of the background field [7] in order to avoid, for sufficiently large values of the physical (scalar) field the kinetic terms with the wrong sign.

II. 'SHIFTED' THEORY PROPAGATORS. UNCONSTRAINED CHIRAL SUPERFIELD POTENTIALS

The chiral superfields Φ_i, $i = 1,2,\ldots,n$ satisfy the differential constraints $\bar{D}\Phi_i = 0$, $D\bar{\Phi}_i = 0$ and it seems difficult to formulate the functional integral over Φ, $\bar{\Phi}$. We may, however, analogous to the case of e.m. field introduce the unconstrained superfield potentials [5),8] S and $\bar{S}$ such that *)

*) We follow the notation of Ref.5. We define $D^2P_2 = D^2$, $d^6s = d^4x\, d^2\theta$, $d^2z = d^4x\, d^2\theta\, d^2\bar{\theta}$.

$$\Phi_i = -\frac{1}{4}\bar{D}^2 S_i \qquad , \qquad \bar{\Phi}_i = -\frac{1}{4} D^2 \bar{S}_i \tag{2.1}$$

It introduces in the theory an additional invariance under the Abelian gauge transformations: $S \to S + \bar{D}F$, $\bar{S} \to \bar{S} + DF$. We may take care of it by adding to the action the following ghost-free gauge-fixing term [9]:

$$I_{G.F.} = \alpha^{-1}\int d^8z\ \bar{S}_i(1-P_1)\Box S_i \tag{2.2}$$

The functional integral may then be formulated easily over S and $\bar{S}$. [9] The perturbation theory performed with $S, \bar{S}$ propagators rather than Φ, $\bar{\Phi}$ ones involves integrals over full superspace as is evident from

$$I_{int} = \frac{1}{3} g_{ijk}\int d^8z\ S_i\left(-\frac{1}{4}\bar{D}^2 S_j\right)\left(-\frac{1}{4}\bar{D}^2 S_k\right) + \text{c.c.} \tag{2.3}$$

The Feynman rules for the vertices may be read from (2.3) on applying Wick's theorem in the conventional way.

The background field method for calculating effective action requires the splitting of each superfield into a classical background piece plus a quantum one. In our context we perform the shifts $\Phi_i \to \Phi_i + C_i$, where C_i are background chiral superfields, $\bar{D}C_i = 0$. In the case of n interacting chiral superfields with the action

$$\int d^8z\ \bar{\Phi}_i Z_{ij}\Phi_j + \left[\int d^6s\ W(\Phi) + \text{c.c.}\right] \tag{2.4}$$

where $W(\Phi)$ is the renormalizable superpotential

$$W(\Phi) = \lambda_i\Phi_i + \frac{1}{2} m_{ij}\Phi_i\Phi_j + \frac{1}{3} g_{ijk}\Phi_i\Phi_j\Phi_k \tag{2.5}$$

the 'shifted' theory contains the following terms:

$$I_0 = \int d^8z\left[\bar{S}_i P_1\Box Z_{ij}S_j - \frac{1}{8}C_{ij}S_i\bar{D}^2S_j - \frac{1}{8}\bar{C}_{ij}\bar{S}_i D^2 \bar{S}_j\right]$$
$$+\alpha^{-1}\int d^8z\ \bar{S}_i(1-P_1)\Box Z_{ij}S_j + \int d^8z\,(J_iS_i + \bar{J}_i\bar{S}_i) \tag{2.6}$$

$$I_{int} = \frac{1}{3} g_{\ell jk}\int d^8z\ \Phi_\ell S_j\left(-\frac{1}{4}\bar{D}^2 S_k\right)$$
$$+\int d^6s\left[\lambda_i\Phi_i + m_{ij}C_i\Phi_j + g_{ijk}C_iC_j\Phi_k + \int d^2\bar{\theta}\ \bar{C}_i Z_{ij}\Phi_j\right]$$
$$+\ \text{c.c.} \tag{2.7}$$

$$I_b = \int d^8z\ \bar{C}_i Z_{ij} C_j + \left[\int d^6s\ W(C) + \text{c.c.}\right] \tag{2.8}$$

Here I_0 is the free action to which we have added the gauge-fixing and an external source term, whereas I_b is the background action. We define the matrix $C = (C_{ij}) = (m_{ij}) + 2(g_{ijk}C_k)$ and have introduced for the latter use the renormalization constants Z_{ij} which are the elements of a positive definite hermitian matrix.

The free 'shifted' theory propagators or Green's functions *) may be derived straightforwardly and we find [5),6)]

$$\Delta^{\bar{S}\bar{S}} = \frac{(\bar{Z}^{-1}C)}{4\Box}\ \bar{D}^2\Delta^{S\bar{S}} \tag{2.9}$$

$$\Delta^{S\bar{S}} = i\left[\alpha\,\frac{(P_2+P_T)}{\Box} + \left\{\Box - Z^{-1}\bar{C}\,P_1\bar{Z}^{-1}C\right\}^{-1}P_1\right]\delta^8(z-z') \tag{2.10}$$

with analogous expressions for Δ^{SS} and $\Delta^{\bar{S}S}$. The term independent of the gauge-fixing parameter in (2.10) may be expressed for the constant background $C_k = a_k + f_k\theta^2$ in a compact form expliciting the poles and θ, $\bar{\theta}$ dependence [5)] which renders the superspace integrations to be performed easily. We find ($Z = 1$)

$$\Delta^{S\bar{S}} = i\,P_1(\Box - \bar{M}M)^{-1}\delta^8(z-z')$$
$$+\ i\,e^{i(\theta\sigma.\partial\bar{\theta} + \theta'\sigma.\partial\bar{\theta}')}\left[A\bar{\theta}^2\theta'^2 + B + G\bar{\theta}^2 + E\theta'^2\right]\delta^4(x-x')$$
$$= i\,P_1\left[P_1(\Box - \bar{M}M)^{-1} + \left\{A\bar{\theta}^2P_1\theta^2 + \frac{B}{16\Box}D^2e^{-2i\theta\sigma.\partial\bar{\theta}}\bar{D}^2\right.\right.$$
$$\left.\left. + G\bar{\theta}^2P_1 + E P_1\theta^2\right\}\Box\right]\delta^8(z-z') \tag{2.11}$$

---

*) They are also useful in case the supersymmetry is spontaneously broken.

where $M = (M_{ij}) = (m_{ij} + 2g_{ijk}a_k)$, $f = (f_{ij}) = 2(g_{ijk}f_k)$ and $C = M + f\theta^2$. The expressions for A, B, G, E are given in Appx.A. The second term in (2.11) arises solely from the explicit supersymmetry breaking terms in (2.6) and clearly vanishes when $f_i = 0$. For the case of a single superfield ($\lambda = 0$)

$$A = \frac{Z^{-3}|f'|^2}{(\Box - Z^{-2}|a'|^2)[(\Box - Z^{-2}|a'|^2)^2 - Z^{-2}|f'|^2]} \quad , \qquad \Box^2 B = Z^{-2}|a'|^2 A \ ,$$

$$G = \bar{E} = \frac{Z^{-3}\bar{f}'a'}{[(\Box - Z^{-2}|a'|^2)^2 - Z^{-2}|f'|^2]} \tag{2.12}$$

where $a' = m + 2ga$, $f' = 2gf$.

III. SUPERFIELD METHOD FOR SUSY EFFECTIVE POTENTIAL

The effective scalar potential may be easily computed using the explicit expressions of the 'shifted' theory superfield propagators given in (2.9) and (2.11). In the superfield tadpole method [5], for example, we are required to compute the tadpole supergraphs for the shifted theory to the desired number of loops. The number of such supergraphs is greatly reduced compared to those encountered in a calculation using the component fields. Moreover, the well-known compensation in an S.S. theory of the higher divergences of boson loops with those arising from fermion loops is already taken care of through the effective superfield propagators derived above. The superfield tadpole method allows us to read off directly the partial derivatives [10] of the effective potential with regard to all the scalar fields present in the theory.

The one-loop correction V_1 to the effective potential for the action in (2.4), to give an illustration, requires the evaluation of a single tadpole supergraph for the 'shifted' theory (see Fig.1) and we find

$$i\Gamma_1^{(1)} = i\,\frac{1}{3}\,3\int d^4\theta\ \tilde{\bar{\Phi}}_\ell(0,\theta,\bar\theta)\left[\mathrm{Tr}\ g_\ell\left(-\frac{1}{4}D^2\right)\Delta^{\bar{S}S}(z,z')\right]_{z=z'} \tag{3.1}$$

where $(g_\ell)_{ij} = g_{\ell ij}$, $\Phi(0,\theta,\bar\theta) = \tilde{A}(0) + \sqrt{2}\,\theta\,\tilde{\psi}(0) + \theta^2\,F(0)$ and a tilde

denotes the Fourier transform. Performing the $\theta, \bar\theta$ integrations we read off from the coefficients of $\tilde{A}(0)$ and $\tilde{F}(0)$ the following partial derivatives [5]:

$$\begin{aligned}\frac{\partial V_1}{\partial \bar{f}_\ell} &= -\frac{1}{2}\mathrm{tr}\ \frac{\partial(\bar{H}H)}{\partial \bar{f}_\ell}\left(I_n - \bar{H}H\right)^{-1}\\ &= \frac{1}{2}\mathrm{tr}\left(\frac{\partial X^2}{\partial \bar{f}_\ell}\right)\left(k^2 I_{2n} + X^2\right)^{-1}\end{aligned} \tag{3.2}$$

$$\begin{aligned}\frac{\partial V_1}{\partial \bar{a}_\ell} &= -\frac{1}{2}\mathrm{tr}\left[\frac{\partial(M\bar{M})}{\partial \bar{a}_\ell}\left(k^2 + M\bar{M}\right)^{-1}\left(I_n - H\bar{H}\right)^{-1} + \frac{\partial(\bar{M}M)}{\partial \bar{a}_\ell}\left(k^2 + \bar{M}M\right)^{-1}\left(I_n - \bar{H}H\right)^{-1}\right]\\ &= \frac{1}{2}\mathrm{tr}\left(\frac{\partial Y^2}{\partial \bar{a}_\ell}\right)\left[\left(k^2 + X^2\right)^{-1} - \left(k^2 + Y^2\right)^{-1}\right]\end{aligned} \tag{3.3}$$

and

$$\begin{aligned}V_1 &= \frac{1}{2}\mathrm{tr}\ \ln\left[I_n - H\bar{H}\right]\\ &= \frac{1}{2}\mathrm{tr}\left[\ln\left(k^2 I_{2n} + X^2\right) - \ln\left(k^2 I_{2n} + Y^2\right)\right]\end{aligned} \tag{3.4}$$

which was also derived by other methods [11]. Here $H = f(k^2 + \bar{M}M)^{-1}$, $\mathrm{tr} \equiv -i\int \frac{d^4k}{(2\pi)^4}\ \mathrm{Tr}$, $\frac{\partial V}{\partial f_\ell} \equiv \left.\frac{\partial V}{\partial F_{c\ell}}\right|_{A_c = a,\ F_c = f}$, etc., where 'c' indicates classical field and we define $2n \times 2n$ matrices X^2, Y^2 by

$$X^2 = \begin{pmatrix} M\bar{M} & f \\ \bar{f} & \bar{M}M \end{pmatrix} \ , \quad Y^2 = \begin{pmatrix} M\bar{M} & 0 \\ 0 & \bar{M}M \end{pmatrix} \tag{3.5}$$

The logarithmic divergence in (3.2) or (3.4) may be handled by employing *)

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*) We may as well use the regularization by means of a cut-off since the divergence is only logarithmic and the finite part of the integrals calculated in the two ways coincide with each other upto finite constants. Higher derivative regularization is advisable for three or higher loop calculations. See, for example, P. West, Higher derivative regularization of supersymmetric theories, CALT-68-1226, 1985. See also Ref.9.

dimensional regularization. Since the coefficient of the divergent integral in (3.4) is $\bar{f}f$ which arises in the kinetic term we need to perform only a wave function renormalization.

The superfield vacuum bubble method, on the other hand, gives rise directly to the effective potential and may sometimes be convenient. For example, at the zero loop we obtain from (2.8)

$$-iV_0 = i\left[\int d^4\theta\, \bar{C}_i Z_{ij} C_j + \int d^2\theta\, W(C) + \int d^2\bar{\theta}\, \overline{W}(\bar{C})\right]$$

$$= i\left[\bar{f}_i Z_{ij} f_j + f_i \frac{\partial W}{\partial a_i} + \bar{f}_i \frac{\partial \overline{W}}{\partial \bar{a}_i}\right] \qquad (3.6)$$

The two-loop contribution similarly requires the computation of the following vacuum bubble supergraphs for the 'shifted' theory (see Fig.2). In the superfield tadpole method we simply have to attach, in all possible ways, an external Φ (or $\bar{\Phi}$) leg to these diagrams.

The renormalized effective potential may be obtained by constructing the counter terms recursively starting from the action given in (2.4) where it is understood to be written in terms of the renormalized quantities (the suffix R being suppressed for convenience). The renormalization constant matrix Z is expanded *) in powers of $\hbar$

$$Z = I + \hbar Z_1 + \hbar^2 Z_2 + \cdots \qquad (3.7)$$

and Z_1, Z_2,... are determined by requiring that the divergences cancel to the order of loops being considered.

The procedure adopted here and in the previous section is clearly adapted for the case when the gauge superfields are also present. However, we obtain quadratic terms of the type $\bar{\Phi} V$ in the free action of the 'shifted' theory. In the presence of the explicitly broken supersymmetry we have not been able to find **) a suitable gauge-fixing condition which may remove such terms and consequently diagonalize the superfield propagators.

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*) We use the units $\hbar = c = 1$. The $\hbar$ is introduced here to keep track of the order of the loop.

**) When only the internal symmetry is broken but not the S.S. a modified gauge-fixing condition was given by B. Ouvrut and J. Wess, Phys. Rev. D25, 409 (1982).

IV. TWO-LOOP EFFECTIVE POTENTIAL FOR THE WESS-ZUMINO MODEL

In the case of a single chiral superfield we obtain at the tree level from (3.6)

$$V_0 = -\left[Z|f|^2 + (ma + ga^2)f + (m\bar{a} + g\bar{a}^2)\bar{f}\right] \qquad (4.1)$$

Writing

$$V_0 = V_0^{(0)} + V_0^{(1)} + V_0^{(2)} + \cdots \qquad (4.2)$$

we find

$$V_0^{(0)} = V_{0R} = -\left[|f|^2 + (ma + ga^2)f + (m\bar{a} + g\bar{a}^2)\bar{f}\right] \qquad (4.3)$$

$$V_0^{(1)} = -\hbar Z_1 |f|^2 \qquad (4.4)$$

$$V_0^{(2)} = -\hbar^2 Z_2 |f|^2 \qquad (4.5)$$

where $V_0^{(0)}$ is the (regular) zero loop contribution to the effective potential while $V_0^{(1)}$, $V_0^{(2)}$,... act as counter terms for the cancellation of the divergent terms in the higher loop contributions.

We find from (3.2) at the one-loop

$$\frac{\partial V_1}{\partial \bar{f}'} = \frac{1}{2}\,\mathrm{tr}\, Z^{-2} f' \Big/ \left[(k^2 + Z^{-2}|a'|^2)^2 - Z^{-2}|f'|^2\right]$$

$$= \frac{-i}{2Z^2}\int \frac{d^4k}{(2\pi)^4}\, \frac{f'}{\left[(k^2 + |a'|^2)^2 - Z^2|f'|^2\right]} \qquad (4.6)$$

where we make the change of variable $k \to Zk$. On performing dimensional regularization we obtain

$$\frac{\partial V_1}{\partial \bar{f}'} = \frac{1}{64\pi^2}\,\frac{1}{Z^2}\, f'\left[-\left(\frac{4}{\varepsilon} + 2 - 2\gamma\right) + \frac{1}{Z|f'|}\left\{(|a'|^2 + Z|f'|)\ln\frac{|a'|^2 + Z|f'|}{\mu^2} - (|a'|^2 - Z|f'|)\ln\frac{|a'|^2 - Z|f'|}{\mu^2}\right\}\right] \qquad (4.7)$$

Writing

$$V_1 = V_1^{(1)} + V_1^{(2)} + \cdots \tag{4.8}$$

we find from (4.7)

$$\frac{\partial V_1^{(1)}}{\partial f'} = \frac{1}{64\pi^2}\left[-\left(\frac{4}{\epsilon}+2-2\gamma\right)+\frac{1}{|f'|}\left(a_+^2 \ln\frac{a_+^2}{\mu^2} - a_-^2 \ln\frac{a_-^2}{\mu^2}\right)\right]f' \tag{4.9}$$

$$\frac{\partial V_1^{(1)}}{\partial f'} = \frac{1}{64\pi^2} Z_1\left[\left(\frac{2}{\epsilon}+\frac{1}{2}-\gamma\right)+\frac{1}{4}\left(\ln\frac{a_+^2}{\mu^2}+\ln\frac{a_-^2}{\mu^2}\right)\right.$$
$$\left. -\frac{3}{4}\frac{1}{|f'|}\left(a_+^2 \ln\frac{a_+^2}{\mu^2} - a_-^2 \ln\frac{a_-^2}{\mu^2}\right)\right]f' , \tag{4.10}$$

where $a_\pm^2 = b^2 \pm |f'|$ and we set $b^2 = |a'|^2$. The expression for the partial derivative $\partial V_1/\partial \bar{a}'$ is also found easily and we find after integrating the partial differential equations

$$V_1^{(1)} = \frac{1}{64\pi^2}\left[-\left(\frac{4}{\epsilon}+3-2\gamma\right)|f'|^2 + a_+^4 \ln\frac{a_+^2}{\mu^2} + a_-^4 \ln\frac{a_-^2}{\mu^2}\right.$$
$$\left. -2b^4 \ln\frac{b^2}{\mu^2}\right] \tag{4.11}$$

$$V_1^{(1)} = \frac{Z_1}{16\pi^2}\left[2\left(\frac{1}{\epsilon}+\frac{1}{2}\right)|f'|^2 - a_+^4 \ln\frac{a_+^2}{\mu^2} - a_-^4 \ln\frac{a_-^2}{\mu^2}\right.$$
$$\left. +2b^4 \ln\frac{b^2}{\mu^2} + \frac{1}{2}|f'|\left(a_+^2 \ln\frac{a_+^2}{\mu^2} - a_-^2 \ln\frac{a_-^2}{\mu^2}\right)\right] . \tag{4.12}$$

They are seen to vanish for $f \to 0$. In order to remove the divergence to first order, e.g. in $(V_0^{(1)} + V_1^{(1)})$ we may choose for Z_1 the value

$$Z_1 = -\frac{g^2}{16\pi^2}\left(\frac{4}{\epsilon}+3-2\gamma\right) \tag{4.13}$$

The regularized one-loop contribution to the effective potential is then given by

$$V_{1R} = \frac{1}{64\pi^2}\left[a_+^4 \ln\frac{a_+^2}{\mu^2} + a_-^4 \ln\frac{a_-^2}{\mu^2} - 2b^4 \ln\frac{b^2}{\mu^2}\right] \tag{4.14}$$

For the corrections upto two loops we may set $Z = 1$ in the computation of the two-loop vacuum bubbles (Fig.2) (or tadpoles). It is also convenient to integrate by parts over the full superspace integrals and write the contributions in the form

$$\int d^6 s_1\, d^6 s_2 \left(\bar{D}_1^2 \bar{D}_2^2 \Delta^{ss}\right)^3$$
$$\int d^6 \bar{s}_1\, d^6 \bar{s}_2 \left(D_1^2 D_2^2 \Delta^{\bar{s}\bar{s}}\right)^3$$
$$2\int d^6 s_1\, d^6 \bar{s}_2 \left(\bar{D}_1^2 D_2^2 \Delta^{s\bar{s}}\right)^3 \tag{4.15}$$

We find for the divergent terms

$$V_2 = \frac{4g^2}{(16\pi^2)^2}\left\{\frac{1}{\epsilon^2}|f'|^2 + \frac{1}{\epsilon}\left[\left(\frac{3}{2}-\gamma\right)|f'|^2 - a_+^4 \ln\frac{a_+^2}{\mu^2} - a_-^4 \ln\frac{a_-^2}{\mu^2}\right.\right.$$
$$\left.\left. +2b^4 \ln\frac{b^2}{\mu^2} + \frac{1}{2}|f'|\left(a_+^2 \ln\frac{a_+^2}{\mu^2} - a_-^2 \ln\frac{a_-^2}{\mu^2}\right)\right] + \cdots\right\} \tag{4.16}$$

The divergent terms $1/\epsilon$ as well as $1/\epsilon^2$ are removed upto the second order, e.g. in $V_0^{(2)} + V_1^{(2)} + V_2$ if along with (4.13) we choose for Z_2 the value

$$Z_2 = -\left(\frac{4g^2}{16\pi^2}\right)^2\left[\frac{1}{\epsilon^2} + \frac{1-\gamma}{\epsilon} - \frac{1+\gamma}{4} - \frac{\pi^2}{24}\right] \tag{4.17}$$

where $\gamma = 0.5772$ is Euler-Mascheroni constant.

The regularized two-loop contribution to the effective potential is obtained to be

$$V_{2R} = \frac{g^2}{(16\pi^2)^2}\left\{\frac{1}{2}\left(a_+^2 \ln\frac{a_+^2}{\mu^2} + a_-^2 \ln\frac{a_-^2}{\mu^2} - 2b^2 \ln\frac{b^2}{\mu^2}\right)^2\right.$$

$$
-\frac{3-2\gamma}{2}\left(a_+^4 \ln\frac{a_+^2}{\mu^2} + a_-^4 \ln\frac{a_-^2}{\mu^2} - 2b^4 \ln\frac{b^2}{\mu^2}\right)
$$

$$
-\frac{3-2\gamma}{2}b^2\left(a_+^2 \ln\frac{a_+^2}{\mu^2} + a_-^2 \ln\frac{a_-^2}{\mu^2} - 2b^2 \ln\frac{b^2}{\mu^2}\right) - \frac{3}{4}\left(a'^2 f' + a''^2 f'' - 2b^4\right) J(1)
$$

$$
-\frac{1}{8}\left(-2b^2 + a'^2\sqrt{\frac{f'}{f''}} + a''^2\sqrt{\frac{f''}{f'}}\right) a_+^2\, J(a_+^2/a_-^2) - \frac{1}{8}\left(-2b^2 - a'^2\sqrt{\frac{f'}{f''}} - a''^2\sqrt{\frac{f''}{f'}}\right) a_-^2\, J(a_-^2/a_+^2)
$$

$$
-\left(a_-^2 - a'^2\sqrt{\frac{f'}{f''}} - a''^2\sqrt{\frac{f''}{f'}}\right) a_+^2\, J(a_+^2/b^2) - \left(a_+^2 + a'^2\sqrt{\frac{f'}{f''}} + a''^2\sqrt{\frac{f''}{f'}}\right) a_-^2\, J(a_-^2/b^2) \Bigg\}
$$

(4.18)

and $V_R^{eff} = V_{0R} + V_{1R} + V_{2R}$ gives the regularized effective potential up to two loops in the modified minimal subtraction scheme specified by (4.13) and (4.17) where the function J is defined in Appendix B.

Each of V_{0R}, V_{1R} and V_{2R} and their partial derivatives vanish in the limit $f \to 0$ in agreement with the non-renormalization theorem when the supersymmetry is not broken at the tree level. The tree level renormalization constraints

$$
\left.\frac{\partial^2 V_{eff}}{\partial a\,\partial f}\right|_{a=f=0} = m \quad , \quad -\frac{1}{2}\left.\frac{\partial^3 V_{eff}}{\partial a\,\partial f}\right|_{a=f=0} = g \qquad (4.19)
$$

are also unaffected by the radiative corrections showing that the superpotential is not renormalized. However, the wave function renormalization results in

$$
-\left.\frac{\partial^2 V_R^{eff}}{\partial f^*\partial f}\right|_{f=0} = 1 - \left(\frac{g^2}{16\pi^2}\right)\left[3 + 2\ln\frac{b^2}{\mu^2}\right]
$$

$$
\frac{1}{2}\left(\frac{g^2}{16\pi^2}\right)^2\left[3 - 2\gamma - J(1) - J'(1) + \frac{3-2\gamma}{2}\ln\frac{b^2}{\mu^2}\right]
$$

(4.20)

We may also calculate the β-functions. In the context of dimensional regularization the canonical dimension of the superfield Φ is $[\Phi] = \frac{(n-2)}{2}$ while $[g] = \frac{(4-n)}{2} = \frac{\epsilon}{2}$. Introducing the dimensionless renormalized coupling constant, $\tilde{g} = g\mu^{-\epsilon/2}$ we have $g_0 = \mu^{\epsilon/2}\,\tilde{g}(\mu)Z^{-3/2}$ where g_0 is the bare coupling constant. Hence *) to two loops 12)

$$
\beta = \mu\left.\frac{\partial\tilde{g}}{\partial\mu}\right|_{\substack{g_0\\ \epsilon\to 0}} = g\left[\frac{3}{2}\left(\frac{g^2}{4\pi^2}\right) - \frac{3}{2}\left(\frac{g^2}{4\pi^2}\right)^2\right] . \qquad (4.21)
$$

It is worth remarking on the ensuing renormalization constraint (4.20). Its left hand side arises in the kinetic energy term $\sim\bar{\Phi}\Phi$ and a negative value for it, say, for sufficiently large b^2 may lead to the wrong sign for the kinetic energy terms of the physical fields. Since [7] in supersymmetric theories in general we have no independent coupling constant renormalization we cannot rectify the situation. In our case we may, alternatively, determine Z_1 and Z_2 by imposing the following constraint:

$$
-\left.\frac{\partial^2 V_{eff}}{\partial f^*\partial f}\right|_{f=0} = 1 \qquad (4.22)
$$

The renormalization can be successfully performed and we obtain at the background field value we wish to consider

$$
Z_1 = -\frac{g^2}{16\pi^2}\left(\frac{4}{\epsilon} - 2\gamma - 2\ln\frac{b^2}{\mu^2}\right)
$$

$$
Z_2 = -\left(\frac{4g^2}{16\pi^2}\right)^2\left[\frac{1}{\epsilon^2} - \left(\frac{1}{2} + \gamma + \ln\frac{b^2}{\mu^2}\right)\frac{1}{\epsilon} + 2\ln\frac{b^2}{\mu^2} - \frac{1}{2}\ln^2\frac{b^2}{\mu^2}\right.
$$

$$
\left. - \frac{1}{2}J(1) - \frac{1}{4}J'(1) + \frac{1}{8}J''(1) + \frac{5}{4} - \frac{\pi^2}{24}\right] \qquad (4.23)
$$

which depend on μ as well as the background constant b^2. For the effective potential we find now

---

*) $dg_0 = \left.\frac{\partial g_0}{\partial\tilde{g}}\right|_\mu d\tilde{g} + \left.\frac{\partial g_0}{\partial\mu}\right|_{\tilde{g}} d\mu$

The running coupling constant at 1-loop is given by

$$
g^2(\mu) = g_0^2\left[1 - 6\left(\frac{g_0^2}{16\pi^2}\right)\ln\frac{\mu^2}{m^2}\right]^{-1}
$$

$$V'_{1R} = V_{1R} - \frac{1}{64\pi^2}\left(3 + 2\ln\frac{b^2}{\mu^2}\right)|f'|^2$$

$$V'_{2R} = V_{2R} + \frac{g^2}{(16\pi^2)^2}\left[\left(3 - \gamma + \frac{1}{8}J''(1) - \frac{1}{4}J'(1) - \frac{1}{2}J(1) + \frac{5-\gamma}{2}\ln\frac{b^2}{\mu^2} + \frac{1}{2}\ln^2\frac{b^2}{\mu^2}\right)|f'|^2\right.$$

$$\left. - \frac{1}{2}\left(\frac{3}{2} + \ln\frac{b^2}{\mu^2}\right)\left(a_+^4 \ln\frac{a_+^2}{\mu^2} + a_-^4 \ln\frac{a_-^2}{\mu^2} - 2b^2\ln\frac{b^2}{\mu^2} - \frac{1}{2}|f'|\left(a_+^2\ln\frac{a_+^2}{\mu^2} - a_-^2\ln\frac{a_-^2}{\mu^2}\right)\right)\right] \tag{4.24}$$

The effective potential as a function of the physical (classical) field is obtained by eliminating the auxiliary field 'f' using its corrected equation of motion $\partial V_{eff}/\partial f = 0$ since it does acquire radiative corrections. The dependence of 'f' as a function of physical field 'a' as well as the effective potential to one and two loops in the Amati-Chou scheme are plotted in Figs. 3(a),(b), where $X = |a'|/m$, $Y = |f'|/m^2$ and $G \equiv g$. We observe that the radiative corrections do not alter the tree level (Eq.(4.3)) minima, the potential remains positive definite, and supersymmetry is not broken. Around the origin $X \approx 0$ (or $a \approx -\frac{m}{2g}$) there is a region where $Y > X^2$ (or $a_-^2 < 0$) and the effective potential becomes complex for these values. In the minimal subtraction scheme, Figs. 4(a),(b) we find, however, also a multivalent effective potential at two loops.

ACKNOWLEDGMENTS

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APPENDIX A

The expressions for A,B, G and E in (2.11) are [5] $(\partial_\ell \to ik_\ell)$

$$A = (k^2 + \bar{M}M)^{-1} - [k^2 + \bar{M}M - \bar{f}(k^2 + M\bar{M})^{-1}f]^{-1},$$

$$k^2B = \bar{M}(k^2 + M\bar{M})^{-1} f\, G,$$

$$k^2G = [\bar{f}(k^2 + M\bar{M})^{-1}f - k^2 - \bar{M}M]^{-1}\, \bar{f}M(k^2 + \bar{M}M)^{-1},$$

$$k^2E = (k^2 + \bar{M}M)^{-1}\, \bar{M}f\, [\bar{f}(k^2 + M\bar{M})^{-1}f - k^2 - \bar{M}M]^{-1} .$$

APPENDIX B

We collect here some useful integrals including the definition of the function J(x):

$$\int \frac{d^{4-\epsilon}p}{(2\pi)^4}\,\frac{1}{(p^2+m^2)} = -\frac{i\,m^2}{16\pi^2}\left[\frac{2}{\epsilon}+1-\gamma-\ln\left(m^2/\mu^2\right)+O(\epsilon)\right] \tag{B.1}$$

$$\int \frac{d^{4-\epsilon}p}{(2\pi)^4}\,\frac{1}{(p^2+m_1^2)(p^2+m_2^2)} = \frac{i}{16\pi^2}\left[\frac{2}{\epsilon}+1-\gamma-\frac{m_1^2\ln(m_1^2/\mu^2)-m_2^2\ln(m_2^2/\mu^2)}{m_1^2-m_2^2}+O(\epsilon)\right] \tag{B.2}$$

$$\int \frac{d^{4-\epsilon}p}{(2\pi)^4}\,\frac{d^{4-\epsilon}q}{(2\pi)^4}\,\frac{1}{(p^2+m_1^2)((p+q)^2+m_1^2)(q^2+m_2^2)} = -\frac{1}{(16\pi^2)^2}$$
$$\Big\{\frac{2}{\epsilon^2}(2m_1^2+m_2^2)+\frac{1}{\epsilon}\Big[(3-2\gamma)(2m_1^2+m_2^2)-2\left(2m_1^2\ln(m_1^2/\mu^2)+m_2^2\ln(m_2^2/\mu^2)\right)\Big]$$
$$+\left(2m_1^2\ln^2(m_1^2/\mu^2)+m_2^2\ln^2(m_2^2/\mu^2)\right)-(3-2\gamma)\left(2m_1^2\ln(m_1^2/\mu^2)+m_2^2\ln(m_2^2/\mu^2)\right)$$
$$+\left(\frac{9}{2}-3\gamma+\gamma^2+\frac{\pi^2}{12}\right)(2m_1^2+m_2^2)+m_2^2\,J(m_2^2/m_1^2)\Big\} \tag{B.3}$$

$$J(x) \underset{(x\le 4)}{=} -4+2\ln(x)-\frac{1}{2}\ln^2(x)-\frac{\sqrt{\pi}}{2}\ln(x)\sum_{j=1}^{\infty}\frac{\Gamma(j)}{\Gamma(j+3/2)}\left(\frac{x}{4}\right)^j$$
$$-\frac{\sqrt{\pi}}{2}\sum_{j=1}^{\infty}\frac{\Gamma(j)}{\Gamma(j+3/2)}\left(\frac{x}{4}\right)^j\left(\psi(j)-\psi(j+3/2)-2\ln 2\right)$$
$$\underset{(x\ge 4)}{=} \frac{\pi^2}{6}-\left(2+\frac{\pi^2}{3}\right)\frac{1}{x}+\frac{2}{x}\ln(x)\sum_{j=1}^{\infty}\frac{\Gamma(j)}{\Gamma(j+2)}x^{-j}$$
$$+\frac{2}{x}\sum_{j=1}^{\infty}\frac{\Gamma(j)}{\Gamma(j+2)}\left(\psi(j+2)-\psi(j)\right)x^{-j}+\frac{1}{x}\ln^2(x)\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\frac{\Gamma(j+k-1)\Gamma(j+k)}{\Gamma(j)\Gamma(k)\,j!\,k!}\Big(2\psi(j+k-1)$$
$$+2\psi(j+k)-\psi(j+1)-\psi(k+1)-\psi(j)-\psi(k)\Big)x^{-j-k}-\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\frac{\Gamma(j+k-1)\Gamma(j+k)}{\Gamma(j)\Gamma(k)\,j!\,k!}$$
$$\Big[\left(\psi(j+k-1)+\psi(j+k)\right)\left(\psi(j+k-1)+\psi(j+k)-\psi(k)-\psi(k+1)-\psi(j)-\psi(j+1)\right)$$
$$+\left(\psi(j)+\psi(j+1)\right)\left(\psi(k)+\psi(k+1)\right)-\psi'(j+k-1)+\psi'(j+k)\Big]x^{-j-k} \tag{B.4}$$

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FIGURE CAPTIONS

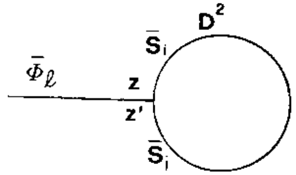
Fig. 1 Tadpole supergraph in one-loop.

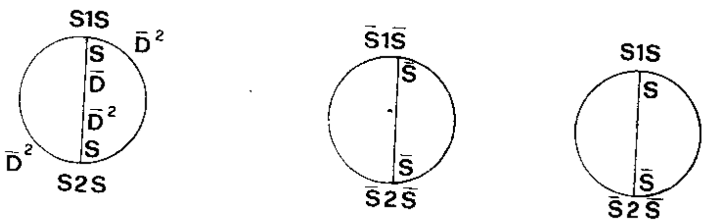
Fig. 2 Vacuum bubble supergraphs needed for two-loop contribution.

Fig. 3 Plot of auxiliary field and potentials against the physical field for (a) $g = 1.57$, (b) $g = 3.14$: (Amati-Chou scheme).

Fig. 4 Plot of auxiliary field and potentials against physical field for (a) $g = 1.57$, (b) $g = 3.14$: (Modified minimal subtraction scheme).

Fig. 5 Plot showing the continuity at $x = 4$ of the function $J(x)$ (and its derivative) as defined in Appendix B.


$\bar{\Phi}_\ell$
z
z'
$\bar{S}_i$
$\bar{S}_j$
D^2


S1S
$\bar{D}^2$
S
$\bar{D}$
$\bar{D}^2$
S
$\bar{D}^2$
S2S
$\bar{S}1\bar{S}$
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$\bar{S}2\bar{S}$
S1S
S
$\bar{S}$
$\bar{S}2\bar{S}$

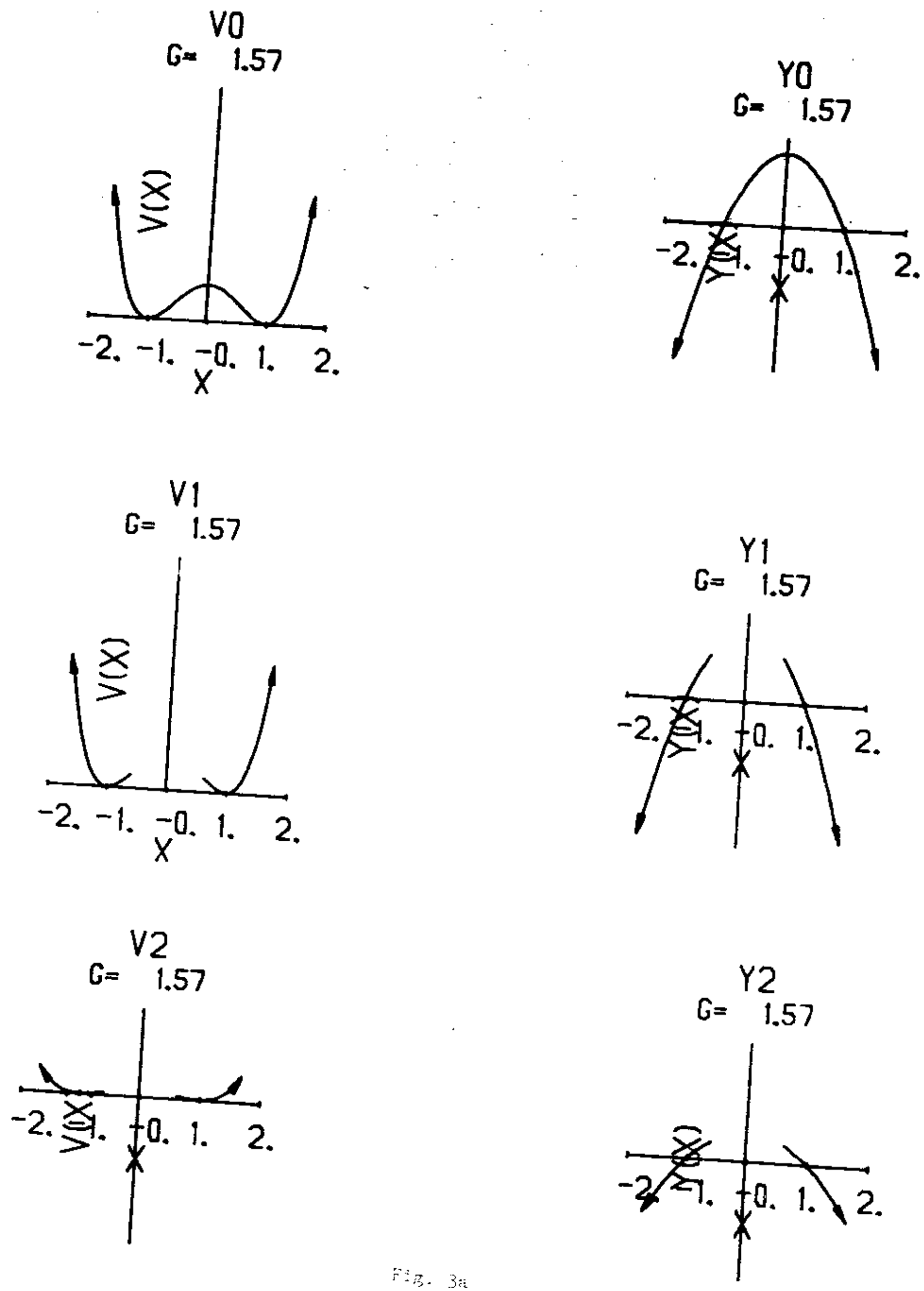
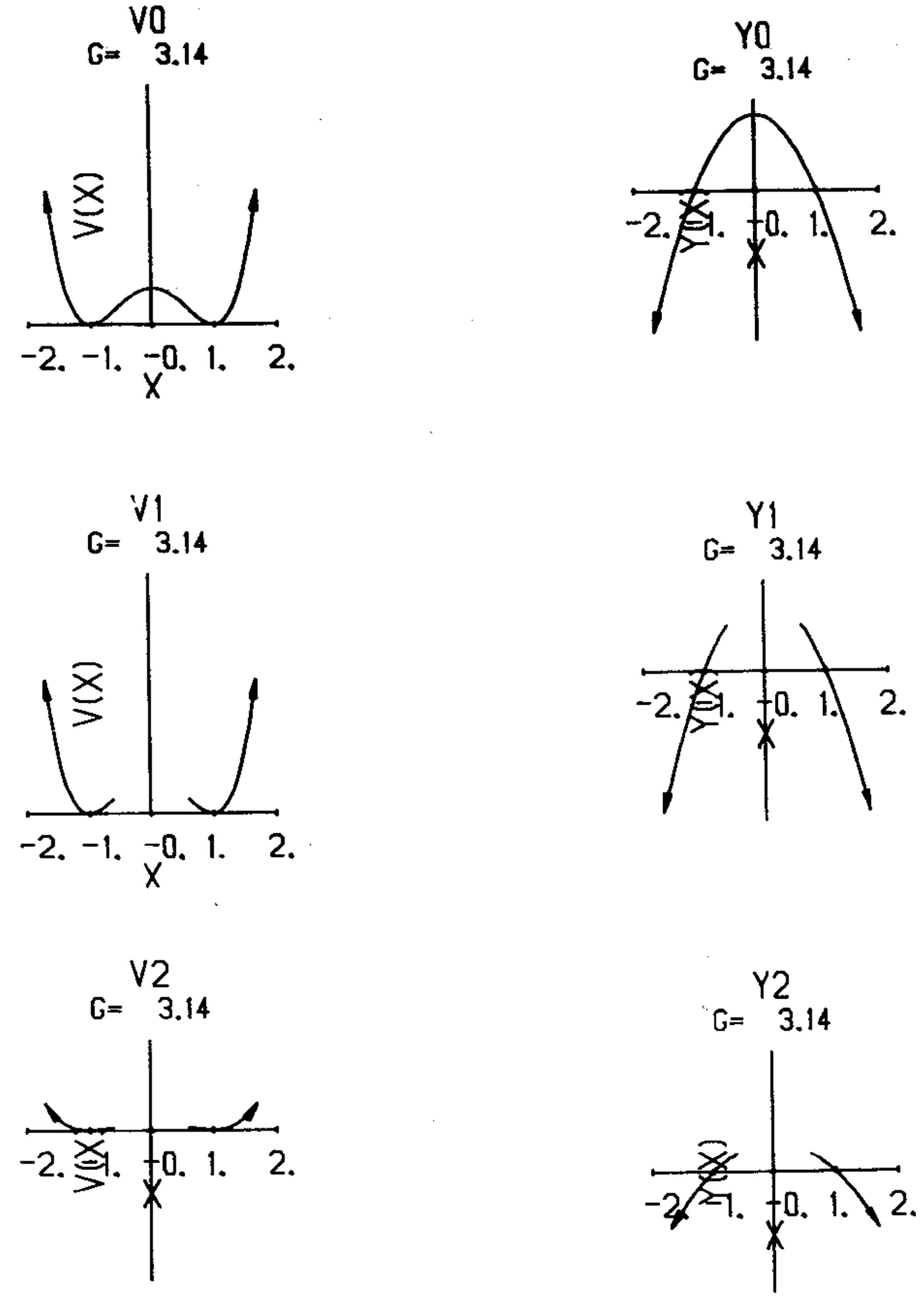

V0
G= 1.57
V(X)
-2. -1. -0. 1. 2.
X
Y0
G= 1.57
V1
G= 1.57
V(X)
-2. -1. -0. 1. 2.
X
Y1
G= 1.57
V2
G= 1.57
Y2
G= 1.57

Fig. 3a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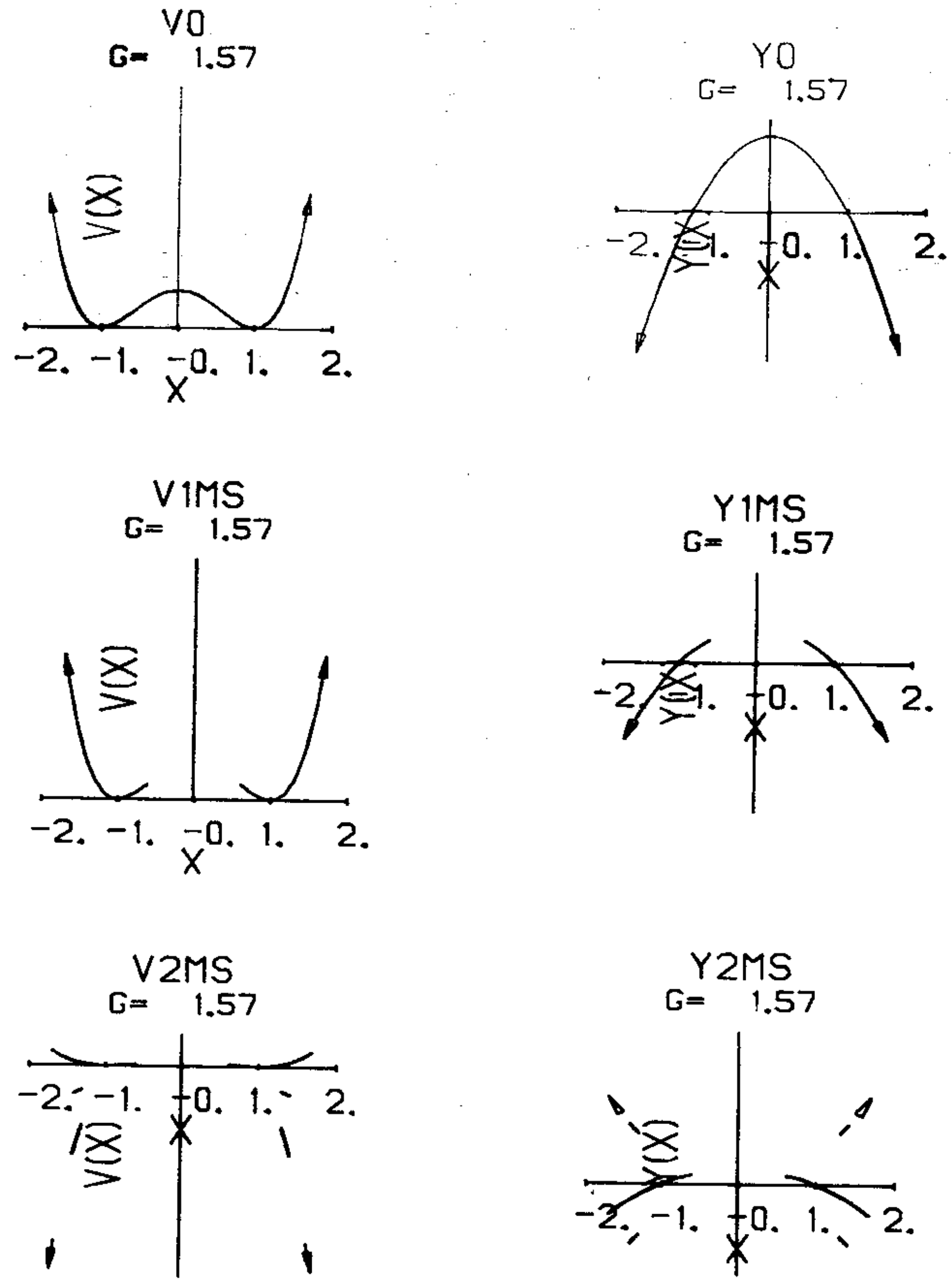
G= 3.14
V(X)
-2. -1. -0. 1. 2.
X
Y0
G= 3.14
Y(X)
-2. -1. -0. 1. 2.
X
V1
G= 3.14
V(X)
-2. -1. -0. 1. 2.
X
Y1
G= 3.14
Y(X)
-2. -1. -0. 1. 2.
X
V2
G= 3.14
V(X)
-2. -1. -0. 1. 2.
X
Y2
G= 3.14
Y(X)
-2. -1. -0. 1. 2.
X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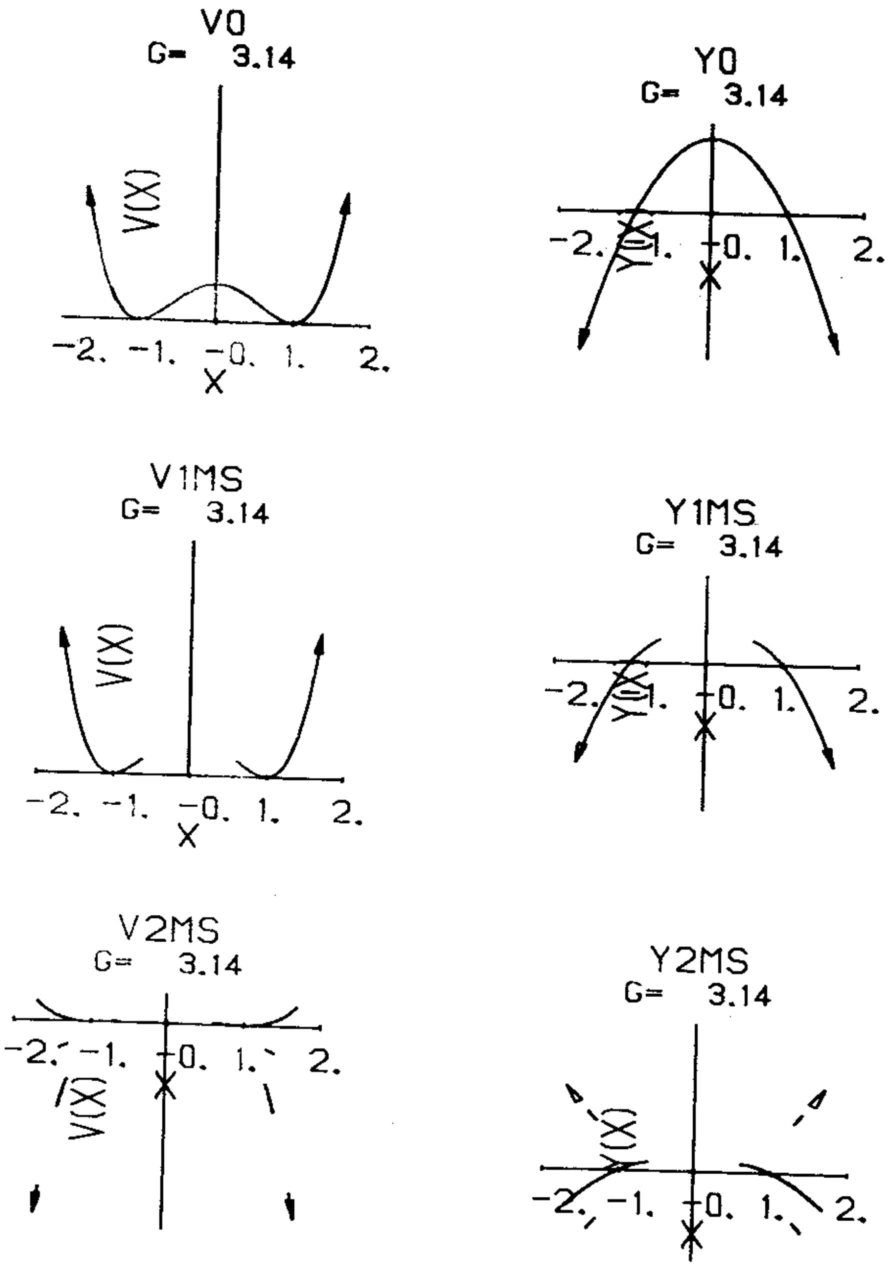
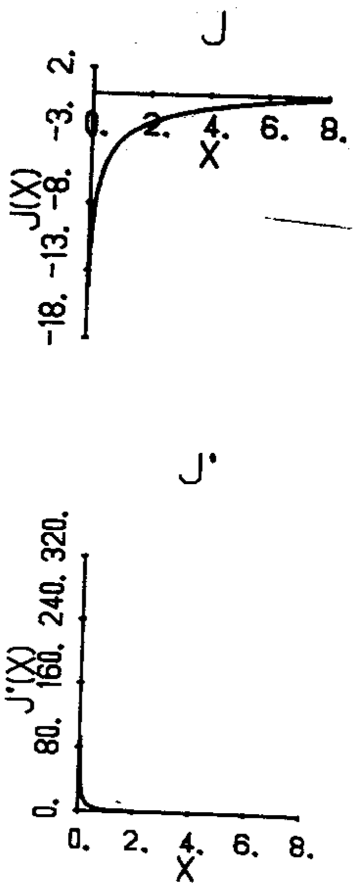
G= 1.57
V(X)
-2. -1. -0. 1. 2.
X
Y0
G= 1.57
Y(X)
-2. -1. -0. 1. 2.
X
V1MS
G= 1.57
V(X)
-2. -1. -0. 1. 2.
X
Y1MS
G= 1.57
Y(X)
-2. -1. -0. 1. 2.
X
V2MS
G= 1.57
V(X)
-2. -1. -0. 1. 2.
X
Y2MS
G= 1.57
Y(X)
-2. -1. -0. 1. 2.
X

Fig. 4a


V0
G= 3.14
V(X)
-2. -1. -0. 1. 2.
X
Y0
G= 3.14
Y(X)
-2. -1. -0. 1. 2.
X
V1MS
G= 3.14
V(X)
-2. -1. -0. 1. 2.
X
Y1MS
G= 3.14
Y(X)
-2. -1. -0. 1. 2.
X
V2MS
G= 3.14
V(X)
-2. -1. -0. 1. 2.
X
Y2MS
G= 3.14
Y(X)
-2. -1. -0. 1. 2.
X


J
J(X)
-18. -13. -8. -3. 2.
0. 2. 4. 6. 8.
X
J'
J'(X)
0. 80. 160. 240. 320.
0. 2. 4. 6. 8.
X