Qualitative Reasoning, Dimensional Analysis and Computer Algebra*

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Introduction

Qualitative Reasoning about physical processes has become very important in various applications involving modeling, design and simulation of devices.

Many approachs to qualitative reasoning about physical processes have appeared in the literature of artificial intelligence (AI). Among them, we can cite *Naive Physics* [1], *Qualitative Process Theory* [2], *Qualitative Physics based in Confluences* [3] and *Qualitative Simulation* [4]. More recently, the Theory of Dimensional Analysis (TDA) has been applied as a supporting technique to *Qualitative Reasoning about Physical Processes*. The main works in this direction have been forwarded by Kokar [5], and in a much formal and systematic approach by Bhaskar and Nigam [6].

The intention of this short progress report is to give an account of a system, in development, which is being particularly designed to compute all the relevant informations to qualitative reasoning about physical processes through the *intra* and *inter-regime* analyses obtained from dimensional analysis [6].

Dimensional analysis

The TDA has its root in the far past work of Fourier [7], who was the first to call attention to the role that dimensions play to Physics. His ideas were later applied by many scientists, in particular by Lord Rayleigh [8], Buckingham [9],

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Riabouchensky [10] and others. The main results of the TDA are summarized in the *Product Theorem*, which establishes that dimensional representations must be multiplicative, and the Π-*Theorem* of Buckingham [9] which states that the physical laws must be complete. This last theorem embodies the *Principle of Dimensional Homogeneity*, which means that all physical formulae must be dimensionally consistent. Further readings can be found in [11].

QDA system

The dimensional analysis of a process can be done (almost) automatically through a computer. The selection of the *performance variables* and of the *basis* are heuristical, nevertheless, once either is given, the algebraic manipulations involved are algorithmic. The symbolic program QDA – Qualitative Dimensional Analyst–, has been particularly tailored to compute all the relevant informations to qualitative reason with dimensional analysis. It is worthwhile mentioning that QDA is a system written in REDUCE [12], a system for symbolic and algebraic manipulation. In what follows we describe the algorithm used to develop QDA :

Step 1: Specification of the physical process.

Step 2: Identification of the variables in the process. Assign their number to n.

Step 3: Identification of the dimensional representation associated to each variable in the process. Assign the number of independent dimensions to d.

Step 4: Construct the dimensional matrix D_M.

Step 5: Determine the rank of the D_M. Assign the rank to r.

Step 6: Determine the number of *regimes* in the process. Assign it to $i = n - r$.

Step 7: Choose the $n - r$ *performance variables* and the *process' basis*.

Step 8: Write and solve the algebraic equations of each *regime*.

Step 9: Write the expression of each Π_i-*regime*.

Step 10: Intra-regime analysis: Determine all partials for the performance variables and analyse their sign.

Step 11: Inter-regime analysis: Identify the contact variables and determine the partials for the performance variables associated to the contact variables and analyse their sign.

The computer algebra system REDUCE is very important in computing the rank of D_M [13], solving the algebraic system of equations and in computing the partials, which are essentially partial derivatives. The qualitative reasoning within and cross the regimes are performed by the QDA system itself. Although it is possible to qualitative reasoning with *ensambles* of regimes, these were not yet included in the QDA system.

Conclusions

We have reported a work in progress which is an interplay of computer algebra, the theory of dimensional analysis and qualitative reasoning about physical processes. As most of the dimensional analysis can be done automatically, being necessary the user intervantion only in the setting of the process variables and in

the choice of either the *performance variables* or the *process dimensional basis*, the system QDA – Qualitative Dimensional Analyst–, is presently being developed to work out all the relevant informations needed to qualitative reasoning by means of dimensional analysis. The QDA system is capable of performing *intra* and *inter-regime* qualitative reasoning. A further extension of QDA to be able to deal with *inter-ensamble* reasoning is under consideration.

The potential applications of QDA to real life problems are not restricted to qualitative reasoning about physical processes only. It may well be applied to many other areas. In fact, one of the major importance of QDA is in the analysis of processes where no *a priori* direct formal knowledge of the laws ruling the devices are available. Of course, qualitative reasoning might not be able to fully respond for the behavior of a process, and so, it does not exclude the association with other approaches, as *quantitative analysis*, for instance. Certainly, the best outcome in the analysis of a device is achieved when all means of investigation are associated together.

In addition, it is worthwhile mentioning here that QDA may be helpful for education purposes, particularly for secundary school students in their learning process of qualitative physical reasoning.

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