Automated Qualitative Reasoning with Dimensional Analysis*

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ABSTRACT

In this paper we discuss qualitative reasoning about processes through dimensional analysis and introduce the system QDR – Qualitative Dimensional Reasoner –, which has been developed to compute all relevant quantities from the dimensional analysis of a process and to perform qualitative reasoning about it through the *intra-regime*, *inter-regime*, *intra-regime-ensemble*, *inter-regime-ensemble* and *qualitative partials* analyses. Several sample of applications in different fields are given using QDR.

1 Introduction

During the past few years *Qualitative Reasoning about physical processes* has become a very important complementary Artificial Intelligence (AI) reasoning methodology as it provides an alternative approach to better understand the behaviour of devices. Various applications involving modeling, design, control and simulation of devices have been done using qualitative reasoning.

Qualitative reasoning is an emerging field of research [9] where the behaviour of devices and/or processes, not necessarely physical, are analysed without paying too much attention on the exact laws describing them. In other words, the analysis is not as accurate as the purely mathematical treatment would be, but is accurate enough to describe consistently the overall behaviour of the process. Qualitative analysis is more concerned with informations than formal resolutions about a process. Inasmuch, in several processes less specific results can reasonably respond for the behaviour of a device where the formal

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laws ruling the device is not known, the computational time spent to get the fine details of the process is fairly large or the qualitative aspects are just the desired goals.

So far several different methods of qualitative reasoning about physical processes have appeared in the literature of AI. Among them, we can cite *Naive Physics* [14], *Qualitative Process Theory* [8], *Qualitative Physics based on Confluences* [16], *Order of Magnitude Reasoning* [26], *Qualitative Simulation* [20], *Comparative Analysis* [30] and *Qualitative Reasoning at Multiple Resolutions* [22]. Several other papers on qualitative reasoning may be found in a newly-published book of collected papers [28], which provides an overall and up-to-date view of the field. More recently, the *Theory of Dimensional Analysis* (TDA) has been applied in AI as a supporting technique to qualitative reasoning about physical processes. The main works in this direction have been forwarded by Kokar in [17] and by Bhaskar and Nigam in [1].

The intention of this paper is to give an account of the system QDR – *Qualitative Dimensional Reasoner* –, which has been particularly designed to automatically compute all the relevant quantities from the TDA and to automate the qualitative reasoning about a process through the *intra-regime*, *inter-regime*, *intra-regime-ensemble*[1] [1], *inter-regime-ensemble* and *qualitative partials* analyses.

The paper is presented as follows: In section 2 we give a short description of the TDA including definitions, the main theorems and references to this theory, where fine details can be found. In section 3 we introduce and discuss various aspects of the QDR system. In particular, the algorithm implemented in it and its reasoning procedure. Section 4 is devoted to show some sample applications of QDR to processes in different fields. Section 5 gives some comments and concludes the paper. In the Appendix A we briefly describe the formulas involved in the regime-calculus and on the various regimes analyses, and finally, in the Appendix B, we give the interaction session of QDR for one of the applications, namely, the pressure regulator.

2 Dimensional analysis

The TDA has its root in the far past works of Newton [23] and of Fourier [10], who were the first to call attention to the role that dimensions play to physics. Their ideas were later applied by many scientists, in particular by Lord Rayleigh [27], Buckingham [3], Riabouchensky [25] and others.

The main results of the TDA have their foundation in the *Principle of Dimensional Homogeneity* (PDH), which states that all physical laws must be dimensionally consistent. In other words, in a formula the dimensional representation of the left-hand-side must be identical to the right-hand-side.

In what follows we state the main theorems of TDA without giving their proofs. However, they can be found in the literature cited in the list of references.

The Product Theorem. Let a secondary quantity be derived from measurements of

[1]This was called *inter-ensemble* analysis in [1].

primary quantities, say α, β, $\gamma, \ldots$. Assuming *absolute significance of relative magnitudes*[2], the value of the secondary quantity is derived as:

$$C_1\,\alpha^a\,\beta^b\,\gamma^c\,\ldots,$$

where C_1, a, b, c, $\ldots$ are constants.

This theorem establishes that dimensional representations must be multiplicative.

The Π-Theorem. Given measurements of physical quantities α, β, $\gamma, \ldots$, such that $\phi(\alpha, \beta, \gamma, \ldots) = 0$ is a complete equation, then its solution can be written in the form $F(\Pi_1, \Pi_2, \ldots, \Pi_{n-r}) = 0$, where n is the number of arguments of ϕ and r is the minimal number of dimensions needed to express the variables α, β, $\gamma, \ldots$. For all i, the Π_i are dimensionless functionals.

The Π-*Theorem*, due to Buckingham [3], provides a nice way to identify the number of dimensionless functionals that characterizes a physical process and in addition it also gives some clues on how to construct the physical law out of the dimensionless functionals as the physical law is contained in the equation,

$$F(\Pi_1, \ldots, \Pi_{n-r}) = 0.$$

According to Buckingham's theorem, an ensemble representation contains all the necessary informations to determine the physical law ruling a physical process. Therefore, any ensemble representation is equivalent to any other, as far as the physical law is concerned.

The Hall's Theorem. Let $\mathcal{S}$ be a finite set of indices, $\mathcal{S} = \{1, 2, \ldots, n\}$. For each $i \in \mathcal{S}$, let $\mathcal{S}_i$ be a subset of $\mathcal{S}$. A necessary and sufficient condition for the existence of distinct representatives x_i, $i = 1, 2, \ldots, n$, $x_i \in \mathcal{S}_i$, $x_i \neq x_j$, when $i \neq j$, is the condition: For every $k = 1, 2, \ldots, n$ and choice of k distinct indices $i_1, \ldots, i_j$, the subsets $\mathcal{S}_{i_1}, \ldots, \mathcal{S}_{i_k}$ contain among them at least k distinct elements.

This theorem guarantees that each regime represents exactly one variable *not* in the basis [1].

For clearness, we shall give here some definitions that will be heavely in use along the paper, and in the appendix A we give a summary of the formulas to obtain the regimes and the various partial analyses.

By *process* we mean all information content describing the system to be investigated. A process is composed of a set of variables obeying the PDH. When a set of variables *does not* fulfil the PDH, we have an incomplete process variables set or an *incomplete specification problem*, otherwise they are simply called *process variables.*

[2]In [2] this means that the ratio between two measurements of quantities is independent of the system of units used. Mathematically, this corresponds to the function that forms the secondary quantities be homogeneous.

The Π-Theorem of Buckingham and Hall's Theorem show that, in a process with n variables we may select p variables of interest. We call these variables, following [1], *performance variables* as long as the remaining r variables satisfy the requeriments to form a *basis*, namely: i) their dimensional representations are linearly independent and ii) all dimensions are included.

An *ensemble* is a set of process variables which has at least one consistent subset of performance variables and *basis variables.* When an ensemble allows m different subsets of consistent performance variables and basis variables, we say that we have m distinct *representations* for the ensemble.

By a *regime*[3] we mean a dimensionless functional found within a representation of an ensemble. According to Hall's Theorem, in a regime only one performance variable appears. It is convenient to take the exponent of the performance variable equal to one.

A regime can be seen as generating a family of hypersurfaces in the process variables space (the process variables are the coordinates). A *critical process hypersurface* [18] is a particular hypersurface (particular value of the regime) where a transition in the process occurs leading to qualitatively different process behaviours.

In an ensemble representation a variable is called an *inter-regime contact variable* when it appears simultaneously in two regimes. This variable makes a bridge between two regimes in the same ensemble representation. They are very important for the inter-regime analysis. For short and compatibility with the definition in [1], we shall refer to these variables simply as *contact variables.*

When two variables of different ensembles are linearly dependent, one can construct a regime within the *process ensemble*[4] coupling these ensembles. This regime is called a *contact* or *coupling regime.* Coupling regimes are very important for the inter-ensemble qualitative reasoning as we will see later.

In a multi-ensemble process the reasoning with variables from different ensembles is possible directly through the coupling regimes (*intra-regime-ensemble analysis*) or in a broader sense through the *inter-regime-ensemble analysis.* An *inter-ensemble contact variable* is a variable that appears in a coupling regime. It plays the analogue role of the contact variable in the inter-regime analysis. These variables form a bridge between regimes from different ensembles.

In an ensemble there might be some variables with the same dimensional representation. When a regime is made out of variables with the same dimensional representation, they are called *simplex regimes.* Otherwise, they are called *complex regimes.* Therefore, coupling regimes are always simplexes.

The number of representations of dimensionless functionals in an ensemble is given by $\binom{n}{r}$, where n is the number of process variables of the ensemble and r is the rank of the dimensional matrix M_D. The number of regimes in any ensemble representation is given by $p = n - r$. This number corresponds to the number of *regime generators* of an

[3]In [1], the distintion between regimes and dimensionless functionals were not very clear made. Clearly, all regimes within an ensemble are dimensionless functionals, but the inverse is *not* true. Dimensionless functionals have been known in the literature as *dimensionless numbers, dimensionless groups, invariants of similitude, similitude numbers* or *similitude modules.*

[4]Process ensemble is the ensemble construct with all the various ensemble variables.

ensemble. Any regime of an ensemble is obtained by products of powers of the generators. In this regard, we say that the regime generators are *linearly independent* (LI) regimes and form a *complete set* of regimes.

Let us call the number of all possible regimes within an ensemble by q. When a regime appears in different ensemble representations, we say that it is an *invariant regime* whose *invariance order* is given by the number of its recurrence. Then, the number of all *distinct* regimes of an ensemble is given by q minus the sum of the invariance order of all regimes[5].

The complete characterization of an ensemble is given by including the regimes of an ensemble representation as additional rows in the dimensional matrix M_D, forming the *extended dimensional matrix* M_E. The full characterization of a process is given by the *process' matrix* M_P, where the columns represent the process variables of each ensemble and the rows their dimensional representations, the ensemble regimes and the coupling regimes.

The importance of dimensionless functionals can be evidenced by their diversified applications in fields like fluid dynamics [6], astrophysics [21], chemistry [4], biological systems [13], biophysical-ecology [11], etc. Some well known examples of dimensionless functionals are: the *Reynold's number*, that occurs in fluid dynamics, indicating when a fluid flow is laminar ($R < 2000$), transitional ($2000 < R < 3000$) or turbulent ($R > 3000$); the *Nusselt's number*, which describes the ratio of the convective conductivity of a surface to the thermal conductivity per unity of dimension; the *Prandtl's number*, describing the relative efficiency of the conducting system for the molecular transport of momentum and energy; the *Grashof's number*, which gives an account of the chimney effect in free convection; and many others dimensionless functionals in other fields as can be seen in ref. [7].

Further readings on TDA can be found in [2], and in a more mathematical approach in [5] and [31].

3 QDR system

The dimensional analysis of a process can be done automatically through a computer. Once the process variables with their corresponding dimensional representations are given, the symbolic/algebraic manipulations involved are purely algorithmic.

The symbolic system QDR – Qualitative Dimensional Reasoner –, has been particularly tailored to compute all the relevant quantities to qualitatively reason with dimensional analysis. It is worthwhile mentioning that QDR is a system written in REDUCE [15], a software with symbolic and algebraic manipulation programming facilities. However, to use QDR, one needs only very little knowledge of REDUCE.

[5]It should be pointed out that, although there is this number of distinct regimes in an ensemble, only p of them are LI. Therefore, any set of p LI regimes of an ensemble is sufficient to fully describe the ensemble.

3.1 The algorithm

In what follows we describe the algorithm used in developing QDR :

Step A: Specification of the process. Input the process variables and their dimensional representations for each ensemble. Determine the number of ensembles and assign it to s.

Step B: **For** $ens := 1 \rightarrow s$ **do**

- **Step B1:** Determine the process variables and their dimensional representations describing this ensemble. Assign the number of them to n and the number of independent dimensions to d.
- **Step B2:** Construct the *dimensional matrix* M_D. Verify the fulfilment of the PDH. Determine the rank of M_D and assign it to r. Determine the number of *regimes* in the ensemble, $n - r$, and assign it to p.
- **Step B3:** Chosen the $n - r$ *performance variables*, check the linear independence of the r left variables and the occurrence of all dimensions to form the *process' basis*. **If** not fulfiled, ask for another choice.
- **Step B4:** Write and solve the system of algebraic homogeneity equations of each *regime* and then write the expression of each Π_p-regime. Write the *extended dimensional matrix* M_E. Look for global contact variables and superfluous variables. In the latter case, ask the user for confirmation.
- **Step B5:** Intra-regime analysis: Determine all *partials* for the performance variables with respect to the basis variables and analyse their signs.
- **Step B6:** Inter-regime analysis: Identify the *contact variables* and determine the partials for the performance variables associated to the contact variables and analyse their signs.
- **Setp B7:** In the *batch-like* mode, generate another representation and **go to** step **B3**. In the *interactive* mode, ask the user whether he wants to run another representation for this ensemble. **If** yes, **go to** step **B3**.

Step C: Intra-regime-ensemble analysis: Identify the *inter-ensemble contact variables* and the *coupling regimes*, determine their partials and analyse their signs.

Step D: Inter-regime-ensemble analysis: Identify the ensemble regimes associated to these contact variables. Determine the partials and analyse theirs signs.

Step E: List the representations so far analysed. Write the *process matrix* M_P. **Stop**.

The algebraic computational power of REDUCE is very important in computing the rank [24] of M_D, solving the algebraic system formed by the homogeneity equations, in computing the partials and in determining the contact variables and the coupling regimes. Observe that the number of calculations involved can become very large, if not impractical to be done by hand. The qualitative reasoning within and across the regimes and ensembles is performed by the QDR system itself.

3.2 The qualitative reasoning

Once we have specified the process variables of each ensemble and their dimensional representation, the system QDR starts computing according to the algorithm above. The whole execution of QDR is done in two main cycles. Firstly, each ensemble is analysed individually in a representation and secondly, the set of ensembles composing the process is analysed.

The specification of the performance variables for each ensemble is automatic in the *batch-like* mode in contrast to the *interactive* mode. In the latter, QDR asks which variables the user would like to choose as the performance variables. It checks if the number of performance variables is consistent. If so, it checks if the remaining variables fulfil the ensemble basis requirements (the linear independence and occurence of all dimensional representation in the basis). Otherwise, in both cases, it returns control to the user to choose a new set of performance variables. Once this is well specified, the system computes all regimes and writes the extended dimensional matrix, M_E.

On the other hand, it might be possible that a process variable is completely irrelevant to the process. In other words, the variable can be thrown out of the process without causing any inconsistency. This *superfluous variable* is easily detected looking at the extended dimensional matrix, M_E (or the process matrix, M_P). It is characterized by having vanishing elements for all regimes.

It might be possible that in an ensemble a non-superfluous process variable can be put aside and the remaining process variables form a consistent sub-ensemble. The sub-ensemble is *process-wise* less informative than the prior ensemble. In other words, the inclusion of the non-superfluous variable in the sub-ensemble will certainly improve the description of the process and consequentely the qualitative reasoning about it. That is what we refer to as *process enrichment*[6].

The qualitative reasoning performed by QDR is based in the various regime and qualitative partials [9] analyses. The intra-regime analysis is easily obtained because it involves only the partials of the performance variables with respect to the basis variables that are present in the regime. In principle all the partials are computed. Nevertheless, not all partials bring new informations about the behaviour of the performance variables with respect to the basis variables.

The system is also able to inform the user how the performance variables do vary with respect to the basis variables within a regime. In other words, it indicates the order (power) of variation (ex. linear, quadratic, cubic, etc.).

In the inter-regime analysis, the qualitative reasoning is done with respect to the performance variables. However, this analysis is only possible when there are contact variables between the regimes. A variable is a *contact variable* between two regimes when it has non-vanishing elements in the extended dimensional matrix, for the regimes in question. Of course, it is necessary only to look for contact variables among the basis variables. Therefore, the maximum number of contact variables between any two regimes is r. When a variable is found to be a contact variable for *all* regimes in an ensemble

[6]Process enrichment should not be misleaded with the incomplete specification problem we have mentioned before.

representation, QDR signalizes it as a *global contact variable*. Global contact variables are those found within the extended dimensional matrix as having non-vanishing elements for all regimes in an ensemble representation. Once the contact variables are identified, the partials are computed and their signs are analysed as for the intra-regime analysis.

To reason across-ensembles, it is necessary to identify the coupling regimes and the inter-ensemble contact variables, among the various ensembles. Coupling regimes are regimes constructed out of two process variables, from different ensembles, which have linearly dependent dimensional representations. The inter-ensemble contact variables are the variables that appear in the contact regimes. Once they have been identified, QDR computes the partials and analyses their signs in a similar way as for the intra-regime and inter-regime analyses (see appendix A).

4 Applications

In this section we give several sample of applications of qualitative reasoning about processes in different fields. All of them were worked out by the QDR system. As a matter of fact, we wish to point out that we have passed to QDR all dimensional analysis examples that appears in the book of Bridgman (see ref. [2]) to be recomputed and then qualitatively analysed.

We stress here that the formulation of the problem to be investigated by QDR is domain specific. In other words, the user should have the necessary knowledge of his field to define the process.

Along the examples we give some comments and indicate how the system behaves step-by-step and on adversities. The statements printed in typewriter letter style are actual extracts from QDR output.

Let us start considering three classical physical processes that appear in the literature of qualitative reasoning and which were analysed in [1], namely: *The motion of a projectile*, *The heat exchanger* and *The pressure regulator*. In the sequel comes, *The RLC circuit*, *The gravitational attraction*, *Material engineering*, *Solar energy* and *Medicine*.

4.1 The (vertical) motion of a projectile

In this example the physical process is the motion of a projectile (see [1], pp 87) when it is shot vertically from the ground. We shall consider the simplest case when no dissipative forces are acting on the particle. The variables describing this motion are: v, the initial velocity; t_1, time of raise; t_2, time of fall; h, the maximum height attained by the projectile; g, the gravitational acceleration.

Thus, informing QDR that the process variables are {T1, T2, G, H, V}, and their dimensional representation are {TIME, TIME, LENGTH$*$TIME^{-2}, LENGTH, LENGTH$*$TIME^{-1}}, respectively, it assigns the number of ensembles, 1, to s, of process variables, 5, to n and the number of independent dimensions, 2, to d. The system is able now to write the dimensional matrix and compute its rank.

They are given, respectively, by,

$$M_D = \begin{pmatrix} & \text{T1} & \text{T2} & \text{G} & \text{H} & \text{V} \\ \text{LENGTH} & 0 & 0 & 1 & 1 & 1 \\ \text{TIME} & 1 & 1 & -2 & 0 & -1 \end{pmatrix}, \quad r = 2.$$

After that, QDR computes the number of regimes, $p = n - r = 3$, and asks the user to choose the performance variables (we are assuming QDR running in its interactive mode, see appendix B). Let us assume that the user has chosen them as follows: {T1, T2, H}. If the number of performance variables chosen by the user is less than the number p of regimes, the system informs the user about and returns control to a new choice of performance variables.

According to the choice of performance variables, QDR checks if the remaining variables fulfil the requirements to form the process basis. If not, it points out the reason (which can be either an incomplete dimensional representation or non-independence of the basis variables) and then returns control to the user for a new choice of performance variables. In the case of incompleteness of the dimensionality of the basis, QDR informs which dimension is missing in the basis.

For the choice of performance variables above, QDR computes the basis {G, V}. Then it writes and solves the algebraic system of homogeneity equations for every regime. Now QDR writes down the expressions of each Π_p-regime computed.

The regimes obtained by QDR are:

$$\Pi_1 = \text{T1} * \frac{\text{G}}{\text{V}}, \quad \Pi_2 = \text{T2} * \frac{\text{G}}{\text{V}}, \quad \Pi_3 = \text{H} * \frac{\text{G}}{\text{V}^2}.$$

Notice that the hypersurface $\Pi_1 = 1$ corresponds exactly to the relation used to compute the time of raise with respect to the initial velocity and gravitational constant; and the hypersurface $\Pi_2 = 1/2$, corresponds exactly to the relation used to compute the height reached by the particle with respect to the initial velocity and gravitational constant, when one looks at the physical formulas describing the process.

The extended dimensional matrix is given by:

$$M_E = \begin{pmatrix} & \text{T1} & \text{T2} & \text{G} & \text{H} & \text{V} \\ \text{LENGTH} & 0 & 0 & 1 & 1 & 1 \\ \text{TIME} & 1 & 1 & -2 & 0 & -1 \\ \Pi_1 & 1 & 0 & 1 & 0 & -1 \\ \Pi_2 & 0 & 1 & 1 & 0 & -1 \\ \Pi_3 & 0 & 0 & 1 & 1 & -2 \end{pmatrix}$$

This matrix contains essentially all informations characterizing the ensemble. In fact, as there is only one ensemble, the extended dimensional matrix fully characterizes the process. In this case it coincides with the *process dimensional matrix*, M_P, which is a matrix encompassing all ensembles.

The intra-regime analysis is done computing all partials of the performance variables with respect to the corresponding basis variables. The reasoning is performed according to the sign of the partials. In addition, the system informs how the performance variable varies with respect to the basis variable in question.

The partials obtained are such that:

$$\frac{\partial\ \mathrm{T1}}{\partial\ \mathrm{V}} > 0, \qquad \frac{\partial\ \mathrm{T2}}{\partial\ \mathrm{V}} > 0, \qquad \frac{\partial\ \mathrm{H}}{\partial\ \mathrm{V}} > 0,$$

$$\frac{\partial\ \mathrm{T1}}{\partial\ \mathrm{G}} < 0, \qquad \frac{\partial\ \mathrm{T2}}{\partial\ \mathrm{G}} < 0, \qquad \frac{\partial\ \mathrm{H}}{\partial\ \mathrm{G}} < 0.$$

According to the partials above, the system QDR informs the user that:

```
The performance variable T1 increases with power 1
as V increases.
The performance variable T2 increases with power 1
as V increases.
The performance variable H increases with power 2
as V increases.
The performance variable T1 decreases with power 1
as G increases.
The performance variable T2 decreases with power 1
as G increases.
The performance variable H decreases with power 1
as G increases.
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In more physical words, the qualitative reasoning of QDR to the performance variable T1, for instance, is understood as follows: *The time of raise increases with the increase of the initial velocity of lauching of the projectile.* Or, for the performance variable H, for instance, *The maximum height of the projectile decreases when the gravitational attraction is stronger.* Of course, the reasoning with respect to a variable is made keeping all the other variables fixed (constant).

The sign analysis done above is exactly what has been defined in [8]. In other words, the qualitative derivative (sign) of a quantity Q can be either $Ds[Q] = -1$, $Ds[Q] = 0$, or $Ds[Q] = 1$, meaning respectively, that the quantity Q decreases, is constant or increases with respect to some parameter variation.

A nice feature of the QDR system is to inform the user how the performance variable changes with respect to the basis variable in the regime. This information is implicitly contained in the extended dimensional matrix. For instance, for the regime Π_1, QDR analyses and concludes that T1 varies linearly (power 1) with V and falls to the power -1 with respect to G. From Π_3, that H varies quadratically (power 2) with respect to V and falls to the power -1 with respect to G. This is important in determining the dimensional dependence in the process.

To be able to perform an inter-regime analysis, QDR first identifies all contact variables. The contact variables found by QDR for this example are {V, G}. Once they have been identified, it computes the partials and their signs in a somewhat similar way as done for the intra-regime analysis.

The contact variables between regimes are easily identified looking at the extended dimensional matrix. For instance, to find out the contact variables between regimes Π_1

and Π_2, it is sufficient to identify which variables have non-vanishing elements for both regimes. Of course, they are only possible among the basis variables.

According to the contact variables found, the inter-regime analysis provided by QDR is as follows:

```
The performance variable T1 increases
as the performance variable T2 increases
for the contact variable G.
The performance variable T1 increases
as the performance variable T2 increases
for the contact variable V.
The performance variable T1 increases
as the performance variable H increases
for the contact variable G.
The performance variable T1 increases
as the performance variable H increases
for the contact variable V.
The performance variable T2 increases
as the performance variable H increases
for the contact variable G.
The performance variable T2 increases
as the performance variable H increases
for the contact variable V.
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Similarly to the intra-regime analysis, the physical interpretation of the results above are as follows: *The time of raise of the projectile increases when its time of fall increases* or, *the time of raise of the projectile increases if the maximum height is increased.*

Notice that there are 10 representations of dimensionless functionals[7] for the projectile process. One regime in another representation in the projectile process is $\Pi = \text{T1}/\text{T2}$. By an appropriate choice of a *hypersurface*[8], $\Pi = 1$, we get the classical physical result that states: *the time of raise of the projectile is equal the time of fall.* Unfortunately, the particular value of a critical hypersurface which gives a physically interesting qualitative behaviour transition seems obtainable only through experiments and/or observations.

As there is only one ensemble, QDR concludes saying that:

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Inter-ensemble analysis is NOT possible with 1 ensemble!
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[7]There are $\binom{n}{r}$ dimensionless functionals representations in an ensemble. The number q of all possible regimes in an ensemble is given by the product of the number of representations of the ensemble, by the number of regime generators p; i.e., $q = m\,p$. The number m, of ensemble representations, is bounded in $1 \leq m \leq \binom{n}{r}$.

[8]In [17], regimes are seen as a family of hypersurfaces. An especific value of the regime, where the process changes its qualitative behaviour, is called a critical hypersurface. $\Pi_1 = 1$ is not a critical hypersurface, but the maximal height establishes one.

Process enrichment: The process richness can be improved if we consider the introduction of the *air resistance* to the motion of the projectile. In fact, this was considered in [1].

Let us introduce then two other process variables. The air resistance, R, and the surface area of the projectile, S. The dimensional matrix becomes,

$$M_D = \begin{pmatrix} & \text{T1} & \text{T2} & \text{G} & \text{H} & \text{V} & \text{R} & \text{S} \\ \text{LENGTH} & 0 & 0 & 1 & 1 & 1 & -1 & 2 \\ \text{TIME} & 1 & 1 & -2 & 0 & -1 & -2 & 0 \end{pmatrix}$$

Now $n = 7$, $d = 2$ and $r = 2$, QDR concludes that the number of possible regimes is $p = 5$. Let us include the variables $\{\text{R},\ \text{S}\}$ in the previous list of performance variables. Thus, QDR computes the extra two regimes[9],

$$\Pi_4 = \text{R} * \frac{\text{V}^4}{\text{G}^3}\,, \quad \Pi_5 = \text{S} * \frac{\text{G}^2}{\text{V}^4}.$$

In [1], this generalization was studied not through these regimes but through their product, namely,

$$\Pi_4 * \Pi_5 = \text{S} * \frac{\text{R}}{\text{G}}\,.$$

Actually, this expression is also a regime[10], found in the ensemble representation with performance variables $\{\text{T1},\ \text{T2},\ \text{H},\ \text{V},\ \text{S}\}$ and basis $\{\text{G},\ \text{R}\}$, as well as the following other regimes:

$$\Pi_1' = \text{T1} * \text{R}^{1/4} * \text{G}^{1/4}\,, \quad \Pi_2' = \text{T2} * \text{R}^{1/4} * \text{G}^{1/4}\,,$$

$$\Pi_3' = \text{H} * \frac{\text{R}^{1/2}}{\text{G}^{1/2}}\,, \qquad \Pi_4' = \text{V} * \frac{\text{R}^{1/4}}{\text{G}^{3/4}}\,.$$

The projectile motion revisited: The vertical motion of a projectile can be treated in a physically more elegant approach. This is achieved by a very simple and closer observation of the physical process. The vertical motion is in fact characterized by the raising process and the falling process. In the former[11], the process variables are just the *initial velocity* v, the *gravitational acceleration* g, the *height* h attained by the particle and the *time of raise* t_1. The latter process[12] is described just by the height h, the *time of falling* t_2 and the gravitational acceleration g.

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[9] More than just being dimensionless functionals, as stated in [1], they are in fact regimes in the same representation.

[10] In [1] this regime was called Π_4.

[11] In fact, just dimensionally speaking, the inicial velocity can be droped out and still have one regime. However, physically speaking, this analysis does not make sense as to raise the particle it is necessary an initial velocity to revert the action of gravity.

[12] In this case the final velocity attained by the particle does not need to be included (its initial velocity is zero), but can be introduced as enrichment for the process.

The above considerations immediately suggest that the projectile motion can be considered as a 2-ensemble process.

The regimes found by QDR for the raising ensemble A with performance variables {T1, H}, and for the falling ensemble B with performance variables {T2} are the following:

$$\Pi_{A1} = \text{T1} * \frac{\text{G}}{\text{V}} , \quad \Pi_{A2} = \text{H} * \frac{\text{G}}{\text{V}^2} ,$$

$$\Pi_{B1} = \text{T2} * \frac{\text{G}^{1/2}}{\text{H}^{1/2}} .$$

In this 2-ensemble approach for the projectile motion, how does the falling time t_2 varies with respect to the initial velocity v? Through the regime analysis given so far that is not possible to know. Nevertheless, qualitative reasoning with variables from distinct ensembles, which are not in a coupling regime, is possible. However, to be able to do so, it is necessary to identify the inter-ensemble contact variables. QDR is also able to identify these variables and to perform qualitative reasoning across the ensembles.

The inter-ensemble contact variables found by QDR are {T1, T2}. Now it is possible, for instance, to compute the inter-regime-ensemble partial of t_2 with respect to v and analyse its sign. Therefore, we get the result,

```
The variable T2 increases as the variable V increases.
```

This qualitative reasoning is clear because for larger initial velocities the time of raise increases and from the contact ensemble an increase in the raising time leads to an increase in the falling time. Consequently, the time of falling also increases when the initial velocity increases (*causal propagation*, see [32]).

This example serves to indicate that in the case of processes with large number of variables, there might be possible to treat the whole process in a multi-ensemble approach.

In the 1-ensemble approach the process is seen as a whole and a full account of the various regime analyses seems to produce a complete qualitative reasoning of the process. While in the multi-ensemble approach the process is seen as a collection of small sub-processes and fine details seems to be more conveniently achieved. Nevertheless, to have a complete qualitative reasoning of the process as a whole, we need to extend the concept of inter-ensemble partials to allow partials with respect to inter-regime-ensemble variables, besides the coupling regimes.

Certainly, both approaches have their own advantages and only a closer look into the process can point out which one will best suit for the qualitative analysis required. Computationally, for a process with a large number of variables, the multi-ensemble approach is likely to be more efficient.

For later purpose, we give below the expression of the law in terms of the regimes describing the raising ensemble, which has been obtained according to the variables elimination procedure developed in [3]:

$$\Phi(\Pi_1, \Pi_2) = \Pi_2 - \Pi_1 \left(1 - \frac{\Pi_1}{2}\right) ,$$

which is not linear in the regime Π_1.

4.2 The heat exchanger

In the heat exchanger (see ref. [1], pp. 89) the process variables are: the density of the oil ρ, the heat transfer area A, the velocity of the oil v, the oil temperature at inlet T_{in}, the oil temperature at outlet T_{out} and the termal conductivity of the pipe material k. Thus, if we set to QDR {RHO, A, V, TIN, TOUT, K}, as the process variables, repectively, and their corresponding dimensional representations[13] {MASS∗LENGTH^{-3}, LENGTH^2, LENGTH∗TIME^{-1}, TEMP, TEMP, MASS∗LENGTH∗TIME^{-3}∗TEMP^{-1}}, the system gives

$$s = 1,\ n = 6,\ d = 4.$$

The dimensional matrix is given by,

$$M_D = \begin{pmatrix} & \text{RHO} & \text{A} & \text{V} & \text{TIN} & \text{TOUT} & \text{K} \\ \text{LENGTH} & -3 & 2 & 1 & 0 & 0 & 1 \\ \text{TIME} & 0 & 0 & -1 & 0 & 0 & -3 \\ \text{MASS} & 1 & 0 & 0 & 0 & 0 & 1 \\ \text{TEMP} & 0 & 0 & 0 & 1 & 1 & -1 \end{pmatrix}$$

which has rank $r = 4$. Thus, the number of regimes is $p = n - r = 2$.

Choosing the following as the performance variables, {TOUT, V}, the system computes the regimes,

$$\Pi_1 = \frac{\text{TOUT}}{\text{TIN}}, \quad \Pi_2 = \text{V} * \frac{\text{RHO}^{1/3} * \text{A}^{1/6}}{\text{K}^{1/3} * \text{TIN}^{1/3}}.$$

The regime Π_2 above is in fact the cubic root of the *Clausius number* N_{cl} [7] (the exact form can be found if we choose another ensemble representation).

The extended dimensional matrix is given by,

$$M_E = \begin{pmatrix} & \text{RHO} & \text{A} & \text{V} & \text{TIN} & \text{TOUT} & \text{K} \\ \text{LENGTH} & -3 & 2 & 1 & 0 & 0 & 1 \\ \text{TIME} & 0 & 0 & -1 & 0 & 0 & -3 \\ \text{MASS} & 1 & 0 & 0 & 0 & 0 & 1 \\ \text{TEMP} & 0 & 0 & 0 & 1 & 1 & -1 \\ \Pi_1 & 0 & 0 & 0 & -1 & 1 & 0 \\ \Pi_2 & 1/3 & 1/6 & 1 & -1/3 & 0 & -1/3 \end{pmatrix}$$

Notice that the regime Π_1 is a simplex regime, while Π_2 is complex.

From the intra-regime analysis, QDR concludes that:

```
The performance variable TOUT increases with power 1
as TIN increases.
The performance variable V increases with power 1/3
```

[13] The dimension of temperature will be represented by TEMP, although we are conscious of the problems with the physical and phylosophical nature of temperature.

```
as TIN increases.
The performance variable V increases with power 1/3
as K increases.
The performance variable V decreases with power 1/3
as RHO  increases.
The performance variable V decreases with power 1/6
as A increases.
```

The contact variable found by QDR is {TIN}. Therefore, only one inter-regime analysis is possible. The QDR result is,

```
The performance variable TOUT increases
as the performance variable V increases
for the contact variable TIN.
```

To reason with respect to other process variables, it is necessary either: change the ensemble representation or to reason through the intra-ensemble-representation analysis (see appendix A). For instance, to reason about the outlet temperature of the oil, TOUT, with respect to the thermal conductivity of the pipe material, K, one can choose the performance variables to be instead {TOUT, K}. This leads to another ensemble representation, among a total of 9, which has the following regimes:

$$\Pi'_1 = \frac{\text{TOUT}}{\text{TIN}}, \quad \Pi'_2 = \text{K} * \frac{\text{TIN}}{\text{RHO} * \text{A}^{1/2} * \text{V}^3}.$$

It is now possible to reason with inter-regime analysis as TIN is a contact variable between Π'_1 and Π'_2.

The inter-regime reasoning provides that:

```
The performance variable TOUT decreases
as the performance variable K increases
for the contact variable TIN.
```

Note that in [1] this inter-regime reasoning was performed with regimes from *different* ensemble representations, what is not needed as we have shown above.

4.3 The pressure regulator

In this process there are two ensembles (see [1], pp 91). We shall present them as they are treated by QDR . The input for QDR is a list containing two lists, each representing one of the ensembles. Thus, the process variables of this problem are {{POUT, Q, PIN, AOPEN, RHO}, {X, P, K}}. The number of ensembles is $s = 2$.

Ensemble A: The ensemble A has the following process variables {POUT, Q, PIN, AOPEN, RHO}, corresponding respectively to, *outlet pressure*, *orifice flowrate*, *inlet pressure*, *orifice opening* and *fluid density*. Their dimensional representations are {MASS $*$ LENGTH^{-1} $*$ TIME^{-2}, LENGTH^3 $*$ TIME^{-1}, MASS $*$ LENGTH^{-1} $*$ TIME^{-2}, LENGTH^2, MASS $*$ LENGTH^{-3}}, respectively.

From the informations above, QDR concludes that for ensemble A, $n = 5$ and $d = 3$. The rank of the dimensional matrix is $r = 3$. Thus, QDR concludes that there are only two regimes. Choosing the performance variables as {POUT, Q}, QDR gets the process basis {PIN, AOPEN, RHO}. Thus, the following regimes are computed,

$$\Pi_{A1} = \frac{\text{POUT}}{\text{PIN}}, \quad \Pi_{A2} = \text{Q} * \frac{\text{RHO}^{1/2}}{\text{AOPEN} * \text{PIN}^{1/2}}.$$

The intra-regime analysis is as follows:

```
The performance variable POUT increases with power 1
as PIN increases.
The performance variable Q increases with power 1/2
as PIN increases.
The performance variable Q increases with power 1
as AOPEN increases.
The performance variable  Q decreases with power 1/2
as RHO increases.
```

QDR identifies {PIN} as the only contact variable. Thus, the inter-regime analysis provides,

```
The performance variable Q increases
as the performance variable POUT increases
for the contact variable PIN.
```

Ensemble B: The ensemble B has the following process variables {X, P, K}, corresponding respectively to, *spring displacement*, *pressure on the top of the piston* and *spring elasticity constant*. Their dimensional representations are {LENGTH, MASS $*$ LENGTH^{-1} $*$ TIME^{-2}, MASS$*$TIME^{-2}}, respectively.

The number of process variables in the ensemble B is $n = 3$, and of independent dimensions is $d = 3$. The rank of the its dimensional matrix is found to be $r = 2$. As we have pointed out before, here we see a particular example where the number of independent dimensions is greater than the rank of the dimensional matrix. This is a decorrence of the fact that the dimensions MASS and TIME appear only in the combination MASS$*$TIME^{-2}, which is the dimensional representation of the physical quantity *force*. In fact, as it is a problem of statics, this ensemble could well be described with FORCE playing the role of a fundamental dimension. In this case the ensemble would have $d = r = 2$. Observe that there is no superfluous variable in this ensemble.

Choosing {X} to be the performance variable, QDR obtains the process basis {P, K}. The only regime found by QDR is,

$$\Pi_{B1} = \text{X} * \frac{\text{P}}{\text{K}}.$$

The intra-regime analysis gives,

```
The performance variable X decreases with power 1
as P increases.
The performance variable X increases with power 1
as K increases.
```

Now QDR can proceed with the across-ensemble analysis. The coupling regimes[14] found by QDR are,

$$\Pi_{AB1} = \frac{\text{POUT}}{\text{P}}, \quad \Pi_{AB2} = \frac{\text{PIN}}{\text{P}}, \quad \Pi_{AB3} = \frac{\text{AOPEN}}{\text{X}^2}.$$

As mentioned before, observe that all the coupling regimes above are simplexes.

From the intra-regime-ensemble analysis, QDR concludes:

```
The variable POUT increases with power 1
as the variable P increases.
The variable PIN increases with power 1
as the variable P incrases.
The variable AOPEN increases with power 2
as the variable X increases.
```

The inter-regime-ensemble analysis through the inter-ensemble contact variables {POUT, PIN, P} gives:

```
The variable POUT decreases as X increases.
The variable Q increases as P increases.
The variable PIN decreases as X increases.
The variable Q increases as X increases.
The variable AOPEN decreases as P increases.
The variable AOPEN increases as K increases.
The variable Q increases as K increases.
```

The inter-regime-ensemble qualitative analysis is new and provides a much simpler and shorter way to get the conclusion that the pressure is mantained constant by the device, as can be seen in the following:

[14]These regimes are neither in the representations of ensemble A nor in the representations of ensemble B. Nevertheless, they are regimes in the process ensemble as a whole, i.e., considering A and B as forming an unique ensemble. Notice that Π_{AB2} was not mentioned in [1].

> If the input pressure PIN increases, the output pressure POUT increases from Π_{A1} regime. On the other hand, from the inter-regime-ensemble analysis, the increases in PIN leads to a decrease in the spring displacement X, which leads to a decrease in the orifice flowrate Q. Finally, from the inter-regime analysis (between Π_{A2} and Π_{A1}) that leads to a decrease in the output pressure POUT.

Although at the first moment there seems to be a contradiction, the qualitative analysis should follows the *causal propagation* [32] reasoning, which leads to the correct behaviour of the device as can be seen below.

The first conclusion (increase in POUT) was obtained taking into account just the ensemble A. The second one (decrease in POUT), takes into account the interaction between the ensembles and expresses the *feedback* behaviour of the pressure regulator.

The same conclusion was obtained in [1], but we should point out that here the number of steps to get the result was 30% less than the ones presented there. This is thanks to the inter-regime-ensemble analysis.

The whole dimensional analysis of the pressure regulator can be summarized in the process matrix below.

$$M_P = \begin{pmatrix} & \text{POUT} & \text{Q} & \text{PIN} & \text{AOPEN} & \text{RHO} & \text{X} & \text{P} & \text{K} \\ \text{LENGTH} & -1 & 3 & -1 & 2 & -3 & 1 & -1 & 0 \\ \text{MASS} & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ \text{TIME} & -2 & -1 & -2 & 0 & 0 & 0 & -2 & -2 \\ \Pi_{A1} & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ \Pi_{A2} & 0 & 1 & -1/2 & -1 & 1/2 & 0 & 0 & 0 \\ \Pi_{B1} & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 \\ \Pi_{AB1} & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ \Pi_{AB2} & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ \Pi_{AB3} & 0 & 0 & 0 & 1/2 & 0 & -1 & 0 & 0 \end{pmatrix}$$

4.4 RLC circuit

In this example we consider a well know electric circuit, namely, the *resistor-inductor-capacitor*-RLC circuit. The intention is to illustrate the qualitative reasoning about a process with 3 ensembles. A simpler circuit, the resistor-capacitor-RC circuit, has been qualitatively analysed with dimensional analysis in [1], by means of the continuity approach in [32] and with multiple resolutions' approach in [22].

The RLC circuit consists of a resistor, inductor and capacitor connected in series. Physically, this process can be seen as a capacitor C, with charge Q which creates a difference of potential, V_c. This potential makes the charges to move creating a current i_c, which has to go through the resistor R and inductor L. Let us assume that the current and potential through the resistor are i_r and V_r, and through the inductor are i_l and V_l, respectively. In this regard, the circuit can be seen as composed of 3 devices, which in turn, induces the process to be considered as composed of 3 ensembles.

Despite we are considering 3 ensembles, we believe that in a first attempt to reason about the RLC circuit, anyone would just use *common sense physics*. The visualization of the process in 3 ensembles should not be, in principle, expected.

Ensemble A: The ensemble A is given by the following process variables {DVC, C, IC}, where DVC is the rate of temporal change of the potential, with the respective dimensional representations {POTENTIAL∗TIME^{-1}, TIME∗CURRENT∗POTENTIAL^{-1}, CURRENT}. Notice that instead of using the mechanical system of units (MLTQ), we have chosen the (LTIΦ) system.

QDR identifies $n = 3$ and $d = 3$. From the dimensional matrix the rank is $r = 2$. Thus, there is only one regime ($p = 1$) in this ensemble, which for the performance variable DVC is:

$$\Pi_{A1} = \text{DVC} * \frac{\text{C}}{\text{IC}} .$$

The intra-regime analysis provides:

```
The performance variable DVC decreases with power 1
as C increases.
The performance variable DVC increases with power 1
as IC increases.
```

In the product form (see appendix A), the regime Π_{A1} expresses the *capacitor's law*:

$$\frac{dV}{dt} = \Pi_{A1} \frac{i}{C} .$$

Ensemble B: The ensemble B is given by the following process variables {VR, R, IR}, with the respective dimensional representations {POTENTIAL, CURRENT^{-1}∗POTENTIAL, CURRENT}.

QDR identifies $n = 3$ and $d = 2$. From the dimensional matrix the rank is $r = 2$. Thus, there is only one regime ($p = 1$) in this ensemble, which for the performance variable VR is:

$$\Pi_{B1} = \frac{\text{VR}}{\text{IR} * \text{R}} .$$

The intra-regime analysis provides:

```
The performance variable VR increases with power 1
as R increases.
The performance variable VR increases with power 1
as IR increases.
```

In the product form, the regime Π_{B1} expresses the *resistor's (Ohm's) law*:

$$V = \Pi_{B1} R i .$$

Ensemble C: The ensemble C is given by the following process variables {VL, L, DIL}, where DIL is the rate of temporal change of the current, with the respective dimensional representations {POTENTIAL, TIME $*$ CURRENT^{-1} $*$ POTENTIAL, CURRENT $*$ TIME^{-1}}.

QDR identifies $n = 3$ and $d = 3$. From the dimensional matrix the rank is $r = 2$. Thus, there is only one regime ($p = 1$) in this ensemble, which for the performance variable DIL is:

$$\Pi_{C1} = \text{DIL} * \frac{\text{L}}{\text{VL}} \,.$$

The intra-regime analysis provides:

```
The performance variable DIL increases with power 1
as VL increases.
The performance variable DIL decreases with power 1
as L increases.
```

In the product form, the regime Π_{C1} expresses the *inductor's law*:

$$V = \Pi_{C1}\, L\, \frac{d\,i}{d\,t} \,.$$

The coupling regimes found by QDR are:

$$\Pi_{AB1} = \frac{\text{VC}}{\text{VR}} \,, \quad \Pi_{AB2} = \frac{\text{IC}}{\text{IR}} \,,$$

$$\Pi_{AC1} = \frac{\text{VC}}{\text{VL}} \,, \quad \Pi_{AC2} = \frac{\text{IC}}{\text{IL}} \,,$$

$$\Pi_{BC1} = \frac{\text{VR}}{\text{VL}} \,, \quad \Pi_{BC2} = \frac{\text{IR}}{\text{IL}} \,.$$

From these coupling regimes, we can obtain, for a suitable choice of hypersurfaces ($\Pi_{AB1} = \Pi_{AC1} = \Pi_{BC1} = 1$), the *Kirchhoff's law* for the RLC circuit:

$$i_c = i_r = i_l = i \,.$$

Notice that the *tension distribution law* can be obtained from any two out of the three coupling regimes (Π_{AB2}, Π_{AC2}, Π_{BC2}) according to the following relation:

$$\Pi_{AC2} + \Pi_{BC2} = 1 \,,$$

which leads to

$$V_c + V_r = V_l \,.$$

The remaining across-ensemble analyses are left out for shortness.

This example could have been made in the 1-ensemble approach. Nevertheless, in this circumstance, the capacitor, resistor and inductor laws are not obtained in a unique representation. In fact, to find out these laws one has to go through 60 distinct ensemble representations. On the other hand, the 1-ensemble approach is useful to analyse aspects associated to the process as a whole, as it is shown in what follows.

The oscillating RLC circuit: Suppose that someone has heard that the RLC circuit has a periodic behaviour and decides to qualitatively analyse this oscilating behaviour using QDR. Thus, the user should be able to inform QDR about the variables describing the process.

The simplest approach (naive physics) to solve the problem is to transpose the knowledge acquired in the previous analysis. This would take him to identify the following process variables: the difference of potential VC, the capacitance C, the difference of potential VR, the resistor R, the difference of potential VL, the inductance L, the current I (from Kirchhoff's law) and the period TAU. These quantities have the following dimensional representation, respectively, {POTENTIAL, CURRENT$*$POTENTIAL^{-1} $*$ TIME, POTENTIAL, CURRENT^{-1} $*$ POTENTIAL, POTENTIAL, CURRENT^{-1} $*$ POTENTIAL $*$ TIME, CURRENT, TIME}.

The number of process variables is $n = 8$, the number of dimensions is $d = 3$. QDR gets, for the dimensional matrix, the rank $r = 3$. Thus, there are $p = 5$ regimes. Choosing {VC, VR, VL, I, TAU} as performance variables, the basis becomes linearly dependent.

Therefore, one has to substitute one of the performance variables {VC, VR, VL, I} by either R, L or C. Let us assume that the substitution is done between I and C.

In this new representation one of the regimes obtained is:

$$\Pi_5 = \mathrm{TAU} * \frac{\mathrm{R}}{\mathrm{L}} \,.$$

From the intra-regime analysis of Π_5 we get the well known result for the period of oscilation of a RL circuit (this can be though of as C being negligible):

$$\tau = \frac{L}{R} \,.$$

On the other hand, if the substitutions were done between I and R or L (R or L being negligible, respectively), the results would be:

$$\Pi_5' = \frac{\mathrm{TAU}}{\mathrm{R}^{1/2} * \mathrm{L}^{1/2}} \qquad \text{or} \qquad \Pi_5'' = \frac{\mathrm{TAU}}{\mathrm{R} * \mathrm{C}} \,,$$

corresponding, respectively, to:

$$\tau = \sqrt{LC} \qquad \text{and} \qquad \tau = RC \,.$$

It is worthwhile mentioning that the period of the RLC circuit is a variable that responds to a *global behaviour* of the process. Therefore, it is only possible to reason about the period if we consider the process as a whole. In other words, by means of the 1-ensemble approach mentioned before. On the other hand, the resistor law (*Ohm's law*), the capacitor law and the inductor law are straightforwardly obtained only through the multi-ensemble approach (3-ensembles).

4.5 The gravitational attraction

Let us study now the simple physical system: the gravitational attraction of two bodies. Let the mass of the bodies be m_1 and m_2, respectively, and their displacement be given by ρ. The Newtonian gravitational constant is G and the force between the bodies due to gravitation is F.

These quantities describe the process. Therefore, the process variables are {M1, M2, RHO, G, F}. The number of ensembles is $s = 1$. The number of process variables is $n = 5$. Their dimensional representations are {MASS, MASS, LENGTH, LENGTH^3*MASS^{-1}*TIME^{-2}, MASS*LENGTH*TIME^{-2}}, respectively. The number of independent dimensions is $d = 3$.

The dimensional matrix has rank $r = 3$. Thus, there are only $p = 2$ regimes. If the variables {M2, F} are chosen to be the performance variables, then QDR concludes that {M1, RHO, G} form the process basis.

The regimes computed are:

$$\Pi_1 = \text{F} * \frac{\text{RHO}^2}{\text{G} * \text{M1}^2}\,, \quad \Pi_2 = \frac{\text{M2}}{\text{M1}}\,.$$

From the intra-regime analysis, QDR concludes that,

```
The performance variable F decreases with power 2
as RHO increases.
The performance variable F increases with power 2
as M1 increases.
The performance variable F increases with power 1
as G increases.
The performance variable M2 increases with power 1
as M1 increases.
```

The contact variable is {M1}. Thus the inter-regime analysis is,

```
The performance variable F increases
as the performance variable M2 increases
for the contact variable M1.
```

According to Buckingham's theorem (see section 2), we may write,

$$\Pi_1 + \Pi_2 = 0,$$

which substituting for their expressions gives,

$$\frac{\text{F} * \text{RHO}^2}{\text{G} * \text{M1}^2} + \frac{\text{M2}}{\text{M1}} = 0.$$

Thus, from the equation above, we can conclude that,

$$F = -g\frac{m_1 m_2}{\rho^2}\,,$$

which is exactly the physical *gravitational law* between two bodies of masses m_1, m_2. It is worthwile pointing out that even the correct sign appears in the equation above, stating that the force F is in fact *attractive*.

As a matter of fact, following the procedure contained in Buckingham's theorem and the same choice of performance variables, we have obtained exactly the above regimes departing from the gravitational law, after the process of variables elimination has been finished.

The power×dimension analysis: Observing the gravitational law itself, it is easy to see that MASS comes into the equation through the product of M1∗M2, which makes the dimension of MASS appears quadratically. Nevertheless, at a first glance, it seems that from the dimensional analysis (see regime Π_1) of the process, the force F would depend quadratically on the dimension of MASS, but made out from M1^2. This result seems to be in contradiction with the actual physical law.

To clear up this puzzle, let us consider the gravitational attraction process in another ensemble representation. From the definitions given in previous sections, the number of ensemble representations of this process is $m = 7$ and of invariant regimes is 4, leading to 10 distinct regimes. Thus in the ensemble representation with performance variables {M1, F}, QDR obtains the following regimes:

$$\Pi'_1 = \frac{\text{M1}}{\text{M2}}\,, \qquad \Pi'_2 = \text{F} * \frac{\text{RHO}^2}{\text{G} * \text{M2}^2}\,.$$

Looking at now to the regime Π'_2, the qualitative analysis from QDR would be similar to the previous representation, but now with M2 instead of M1. Therefore, we can see that: from the former representation force would "depend" on M1^2, while from the latter, force would "depend" on M2^2. Following this reasoning, we would conclude that the physical law would have a product of $\text{M1}^2 * \text{M2}^2$, which would give a dimensional dependence on MASS of fourth order. However, as we have mentioned before, the power indicates how the dimensional representation of the respective process variable should come in in the process' law, rather than on the process variable dependence.

Therefore, the qualitative analysis is correct and the misconcept has been solved. This may not cause surprise because the reasoning of QDR is based on the dimensional analysis of the process and not on the process variables themselves.

According to this result and to the partial analyses of both representations, it is straightforward to see that M1 and M2 can only appear as a simple product in the gravitational attraction law. This particular reasoning requires an *intra-ensemble-representation* (see appendix A) analysis which is presently outside the scope of QDR.

Satellite motion: As an example of process enrichment, let us consider now the *satellite motion* problem (circular motion) and try to qualitatively reason about the period of the satellite (m_2) around the earth (m_1). We shall consider the simplest case, when the masses of the bodies are point-like and the satellite is moving around the earth (fixed point).

Thus, we have a new process variable, say τ, as the period of motion, which should be included in the process variables list, as well as its dimensional representation in the dimensional representations list.

The number of process variables is now $n = 6$ and the rank of the dimensional matrix is still $r = 4$. Notice that TAU is not a superfluous variable and the extended ensemble has no incomplete specification problem.

Including TAU in the list of performance variables, QDR gets the following regime:

$$\Pi_3 = \mathrm{TAU} * \frac{\mathrm{M1}^{1/2} * \mathrm{G}^{1/2}}{\mathrm{RHO}^{3/2}} .$$

The intra-regime analysis is:

```
The performance variable TAU increases with power 3/2
as RHO increases.
The performance variable TAU decreases with power 1/2
as M1 increases.
The performance variable TAU decreases with power 1/2
as G increases.
```

The first two statements above are just saying that: *The period of the satellite increases when its distance from the earth increases*, and *The period decreases if the mass of the earth increases* (the satellite has to move faster, otherwise it would fall down on the earth).

For shortness, we shall leave out the inter-regime analysis performed by QDR.

4.6 Material engineering

Let us consider now an application, in the material engineering field of elasticity, given in Bridgman (see ref. [2], pp. 67). In this application QDR will reason about the stiffness of a beam according to changes in its dimensions and under changes in the material elasticity.

Assuming a beam of retangular shape made out of an isotropic material whose elasticity parameters are Young's modulus E and shear modulus μ, with length l, breadth b, thickness d and stiffness S with the following dimensional representations {FORCE $*$ LENGTH^{-2}, FORCE $*$ LENGTH^{-2}, LENGTH, LENGTH, LENGTH, FORCE $*$ LENGTH^{-1}}, QDR computes for the performance variables {S, B, D, MU}, the regimes

$$\Pi_1 = \frac{\mathrm{S}}{\mathrm{E} * \mathrm{L}} , \quad \Pi_2 = \frac{\mathrm{B}}{\mathrm{L}} , \quad \Pi_3 = \frac{\mathrm{D}}{\mathrm{L}} , \quad \Pi_4 = \frac{\mathrm{MU}}{\mathrm{E}} .$$

The relevant intra-regime analysis are:

```
The performance variable S increases with power 1
as L increases.
The performance variable S increases with power 1
as E increases.
```

From the intra-regime analysis, it is clear that the stiffness becomes stronger when both the length l and/or the Young modulus E increases.

The relevant inter-regime analysis are:

```
The performance variable S increases
as the performance variable B increases
for contact variable L.
The performance variable S increases
as the performance variable D increases
for contact variable L.
The performance variable S increases
as the performance variable MU increases
for contact variable E.
```

According to the inter-regime analysis, the stiffness of the beam also increases if the breadth b, the thickness d or the shear modulus μ are increased.

In this example it was assumed that the beam is under a longitudinal compression. However, an interesting qualitative reasoning about the stiffness of a beam would be to consider the case when the beam is supported at the ends. This would lead to the study of its maximum deflection. This point would establish a critical hypersurface to the problem as above it the process would be qualitatively different (it corresponds to the rupture of the beam).

The reader may notice that the formula approach of [2] is unable, using TDA, without further physical knowledge, to qualitatively reason about the stiffness variation with respect to the other process variables. In fact, this is a crucial difference between the formula driven approach and the regime/partial approach presented here. The latter is for qualitative analysis purposes richer than the former.

4.7 Solar energy

One of the most effective energy conservation devices in the field of solar energy applications is the *heat pumps* [29]. In this section we shall consider the vapor-compression heat pump, which has the primarily objective of raising the temperature of the working fluid. Therefore, it is very important, for various reasons, to work out and analyse the behaviour of different working fluids under changes in their condensing and evaporating pressure P_c, P_e and variation of temperature ΔT $(T_c - T_e)$, to the maximum temperature T^* where the intermolecular attractive potencial is maximum, as well as with respect to their gas constant R and enthalpy h.

The dimensional representations of these quantities are given, respectively, by {PRESSURE, PRESSURE, TEMP, TEMP, ENERGY $*$ TEMP $*$ MASS^{-1}, ENERGY $*$ MASS^{-1}}. The reader might have noticed that here PRESSURE and ENERGY are playing the role of fundamental dimensions.

Therefore, the number of process variables are $n = 6$, the number of independent dimensions are $d = 4$. QDR computes $r = 3$. Here too, the number of independent dimensions is greater than the rank of the dimensional matrix.

Similarly to the previous cases, QDR computes the following regimes for the performance variables {PC, DELTAT, H}:

$$\Pi_1 = \frac{\text{PC}}{\text{PE}} \quad , \Pi_2 = \frac{\text{DELTAT}}{\text{TSTAR}} \quad , \Pi_3 = \frac{\text{H}}{\text{TSTAR} * \text{R}}.$$

One interesting behaviour coming out from the intra-regime analysis is,

```
The performance variable H increases with power 1
as TSTAR increases.
```

From the inter-regime analysis we get,

```
The performance variable DELTAT increases
as the performance variable H increases
for contact variable TSTAR.
```

Notice that as $\Delta T = T_c - T_e$, the sign of the inter-regime partial may change, becoming negative. According to [29], the properties of the working fluid vary extensively for this case. Therefore, the temperature where the fluid changes its qualitative behaviour defines a critical hypersurface. One nice feature of QDR as a symbolic system is that it is able to perform symbolic partial derivative and so, it would be possible to reason particularly with respect to T_c or T_e.

Concerning the heat pump working fluid in solar energy, an important dimensionless functional is written with the above regimes as follows:

$$\alpha^* = \frac{\Pi_2 \, \Pi_3}{\Pi_1} = \left(\frac{\Delta T}{T^*} \frac{h}{R T^*} \right) / \left(\frac{p_c}{p_e} \right).$$

This dimensionless functional, α^*, is the *heat pump performance parameter* (see [29], pp. 43) extensively used to compare the behaviour of different working fluids in solar energy. Several values of α^* has been presented in [29] for various working fluids.

4.8 Medicine

Let us consider now an example from medicine, namely, the determination of the *cardiac output* (see ref. [12], 576). The indicator-dilution method has been applied as one of the approachs to measure the cardiac output by measuring the blood flow through the heart. This method essencially measures the dilution/concentration of a contrast substance injected in the blood.

Assuming that a vain has a cilinder-like shape and that a soluble and detectable indicator is rapidly introduced into the vain and other medical considerations, a concentration detector placed slightly downstream can measure the concentration of the indicator injected.

Thus, if the mass of the contrast substance injected is m, its concentration at downstream is c, the dilution time is t and the blood flow is ϕ, then we can qualitatively reason about the flow in terms of the concentration. Their dimensional representation are, respectively, {MASS, MASS∗VOLUME^{-1}, TIME, VOLUME∗TIME^{-1}}. Notice that here the dimensional representation of volume (LENGTH^3) is taking as one of the fundamental dimensions to the process.

There are $n = 4$ process variables and $d = 3$ dimensions. QDR determines the rank of the dimensional matrix $r = 3$. Thus, the number of regimes is $p = 1$. Choosing the performance variable to be {PHI}, the only regime in this representation is,

$$\Pi_1 = \text{PHI} * \frac{\text{M}}{\text{T} * \text{C}} .$$

The intra-regime analysis provides:

```
The performance variable PHI increases with power 1
as M increases.
The performance variable PHI decreases with power 1
as C increases.
The performance variable PHI decreases with power 1
as T increases.
```

From the intra-regime analysis above it is easy to see that if the device measures at downstream a stronger concentration, this indicates that the cardiac output (blood flow) is slower in contrast to the case of a weaker concentration, which means a faster blood flow.

It is common to measure the cardiac output in *liters/min*, the mass of the indicator in *grams*, the time of dilution in *seconds* and the concentration in *grams/liter*. Although these units of measurements are not the same system (min:second), the qualitative reasoning is not affected. Clearly, the quantitative reasoning has to take that into account through a scale factor of 60. This fact is a decorrence of the invariance of the physical laws under rescaling of units.

5 Conclusions

In this paper we have presented and discussed the main features of QDR – Qualitative Dimensional Reasoner –, a system particularly designed to automate the qualitative reasoning about processes, based in the Theory of Dimensional Analysis, in the concepts of intra-regime, inter-regime, intra-regime-ensemble, inter-regime-ensemble and qualitative partials analyses.

As the dimensional analysis of a process is algorithmic, the system QDR works out all the relevant informations needed to qualitatively reason by means of dimensional analysis. Most of the calculations required are straightforward but lengthy and tedious.

The QDR system is capable of performing intra-, inter-regime, intra- and inter-regime-ensemble qualitative reasoning. For the inter-regime, the inter-regime and intra-regime-ensemble analyses, a constructive way has been presented here (see appendix A) to find out the contact variables, the coupling regimes and inter-ensemble contact variables.

In addition, QDR provides the dimensional matrix M_D, extended dimensional matrix M_E, which fully characterizes the ensembles, and the process matrix M_P, which gives a complete account of the dimensional analysis of a process. An important point to be stressed is that QDR is very fast (the elapsed CPU time for the pressure regulator process example is 50219 ms). Therefore, it allows an overall computation of regimes and their analyses for all ensemble representations.

QDR may run in two distinct modes, namely, the *interactive* mode or the *batch-like* mode. In the former, the QDR demands that the user chooses the performance variables while in the latter, it runs automatically all possible choices of performance variables (see appendix B).

To develop the QDR system we have made use of the symbolic software REDUCE, which is a powerful language incorporating symbolic and algebraic manipulations facilities. The algebraic power of REDUCE is very important in computing the rank of the dimensional matrix, solving the algebraic system of homogeneity equations, in determining the contact variables and in determining the coupling regimes. In principle, one could have made, with greater effort, these calculations by means of a numerical language like FORTRAN or PASCAL. However, REDUCE is far more convenient for the pure symbolic manipulations. Although QDR has been written in REDUCE, very little and simple knowledge of it is necessary to be able to profit from QDR.

In some extent, the QDR system provides a way to solve the *complete relevance problem* and the *relationship problem* possed in [17]. The *semantics problem* is not solved due to its specific nature. Nevertheless, QDR helps the computations involved in determining the critical hypersurfaces and in generating all landmark points given an initial landmark point. It is also able to identify the superfluous variables and the global contact variables of the process.

The sample of applications presented were particularly chosen to show some important aspects of the qualitative reasoning. The first three examples describe pure physical processes. Nevertheless, they show very important features of qualitative reasoning with dimensional analysis like, how to reason with intra-regime, inter-regime, intra-regime-ensemble and inter-regime-ensemble partials, the use of temperature as a fundamental dimensions, the importance of finding critical hypersurfaces (landmark points), the enrichment of a process, etc. In particular, in the pressure regulator example, the inter-regime-ensemble analysis has shown to be very important in obtaining the conclusion that the pressure is in fact kept constant when the input pressure is changed. These examples also clear some minor misleadings found in [1] and exhibit in practice most of the new concepts defined here.

The RLC circuit example shows a very important aspect on the qualitative reasoning about a process. Firstly, it is shown in the multi-ensemble approach to the process and secondly, in the 1-ensemble approach. In the former, each regime gives essentially the Ohm, the inductor and the capacitor laws for the circuit. Also, from the intra-regime-ensemble analysis, by an appropriate choice of hypersurfaces, the Kirchhoff's law is obtained and some regime combinations provides the tension distribution law. In contrast, global behaviour of the circuit seems only transparent throughout an 1-ensemble analysis, as was shown for the period of oscilation of the RLC circuit.

The gravitational attraction example has shown that the actual law ruling the process may be found from the regimes, despite of the non-constuctiveness of Buckingham's theorem. In addition, a discussion about the dimensional power dependence of a variable in the process has been addressed and it is shown that a simple process enrichment lead us to reason about the period of a satellite motion around the earth.

In the material engineering example, the stiffness of a beam is qualitatively analysed, driving our particular attention to the fact that relevant quantities to the process have similar dimensional representation, although neither of them were superfluous to the process. Additionally, it was used to point out that the formula driven approach is qualitatively weaker than the regime approach applied here.

The solar energy example is very interesting because it leads to the definition of an important dimensionless functional currently used in the research of working fluids for heat pumps in solar energy.

Finally, the cardiac output, in the medicine example, shows that qualitative reasoning about processes with regime dimensional analysis is independent of the system of units adopted, as well as scale independent.

As a sign of QDR's reasoning capability, we have made it work out all the examples (more than 20) from Bridgman's book [2] for every ensemble representation. One of them is the "Material Engineering" example shown in section 4.6.

The potential applications of QDR to real life problems are not simply restricted to qualitative reasoning about physical processes only. It may well be applied to many other areas as we have shown throughout a number of sample of applications. In fact, one major importance of QDR is in the analysis of processes where no *a priori* direct formal knowledge of the laws ruling the devices are available.

One shortcoming of this qualitative reasoning approach is that *not* all regime analyses provide an useful, and sometimes meaningful, information about the process' behaviour. For instance, certain partials are sometimes meaningless (see the partial for regime Π_2 in the gravitational attraction example) and the power indicating the functional dependence of a variable does not always matchs the correct value as indicated by the actual physical law. Nevertheless, in the latter case, the dimensional dependence is correctly obtained (see also comments in the gravitational attraction example).

Of course, qualitative reasoning might not be able to fully respond for the behaviour of a process, and so, it does not exclude the possibility of association with other approaches available. Likely, the best outcome in the analysis of a device is achieved when the means of investigation are associated together.

Several aspects of qualitative reasoning about physical processes and comparison with some other approachs have been left out as they were not the primary objective of this paper. Nevertheless, many interesting points and relationships with other approachs have been nicely addressed in the papers of Kokar [19] and Bhaskar and Nigam [1]. An overview on the present status of qualitative reasoning about physical processes can be found in [28].

We believe that several fields, where the number of process variables is fairly large and/or no formal law exist to describe the system as a whole, can be qualitatively analysed with the aid of QDR. As possible candidates we can mention metheorology, power plant, economy, ecology, etc. In particular those non-physical fields where the notion of intransmutable dimensions can be setup.

In addition, it is worthwhile mentioning here that QDR may be employed for education purposes helping students on their learning/explaining process of qualitative reasoning about simple systems found in everyday life and in checking the *Principle of Dimensional Homogeneity* of physical laws. It is our hope that QDR be useful also for building intelligent tutorial systems and for practical applications in control systems, process engineering, decision making and other technological areas.

Appendix A

In this appendix we give a short account of the regime calculus and of the various regime and ensemble analyses. Part of the material presented in this appendix has been borrowed from the paper of Bhaskar and Nigam [1].

Regime calculus: Let us call the set of process variables by $\{v_1, v_2, \ldots, v_n\}$. According to Buckinham's and Hall's theorems, we may select out of the set of process variables $p = n - r$ performance variables, where r is the rank of the dimensional matrix M_D. Let us denote the p performance variables by $\{y_1, y_2, \ldots, y_p\}$ and the remaining r variables, which form the basis variables by $\{x_1, x_2, \ldots, x_r\}$.

The regime Π for the performance variable, say $\{y_i\}$, is given by,

$$\Pi_i = y_i \times (x_1^{\alpha_{i1}}\, x_2^{\alpha_{i2}}\, \ldots\, x_r^{\alpha_{ir}})\,,$$

where the coefficients α_{ij}, $i = 1, \ldots, p\,, j = 1, \ldots, r$ are such that Π_i is a dimensionless functional. They are the solution of the system of algebraic homogeneity equations for the dimensional representation of the corresponding performance variable and basis variables.

From the above expression of a regime, we can write the performance variable in the form

$$y_i = \Pi_1 \times (x_1^{-\alpha_{i1}}\, x_2^{-\alpha_{i2}}\, \ldots\, x_r^{-\alpha_{ir}})\,.$$

When a performance variable is written in the form above we say it is written in its *product form.* (The product form would be different if the coefficient of the performance variable was not suitably made equal to 1.)

Regime analysis: The qualitative reasoning is obtained through the various *regime analyses* and the sign of the *partials.*

The *intra-regime analysis* is done by computing the partial of a performance variable with respect to a basis variables, keeping the remaining basis variables fixed. This procedure is given by,

$$\frac{\partial\, y_i}{\partial\, x_j} = -\alpha_{ij}\, \frac{y_i}{x_j}.$$

The *inter-regime analysis* is done between two performance variables from regimes of the same ensemble represntation. For that, it is necessary to find out first the so called *contact variables* among the various regimes.

The contact variables are basis variables that appear simultaneously in 2 regimes. Let us denote a contact variable by x_c. Thus, the inter-regime partial is given by,

$$\left[\frac{\partial\, y_i}{\partial\, y_j}\right]^{x_c} = \frac{\alpha_{ic}}{\alpha_{jc}}\,\frac{y_i}{y_j}\,,$$

where the symbol $]^{x_c}$ denotes that the partial is computed through contact variable x_c.

The *intra-regime-ensemble analysis*[15] is done with regimes formed from dimensionally linearly dependent (DLD) variables from distinct ensembles, but within a fixed ensemble representation.

In order to reason with intra-regime-ensemble analysis, it is necessary to identify the *coupling* or *contact regimes.* This is easily done looking for dimensionally linearly dependent variables across the ensembles. Thus, the coupling regime is written by,

$$\Pi_{ABk} = \frac{v_{Ai}}{v_{Bj}^{\beta_{ij}}}\,,$$

where the index ABk denotes the k-th coupling regime between the ensembles A and B, v_{Ai}, v_{Bj} are two linearly dependent variables and β_{ij} are coefficients reflecting the dimensional non-linearity of the variables, given by

$$[v_{Ai}] = [v_{Bj}]^{\beta_{ij}}\;.$$

The partial of the variable v_{iA} with respect to v_{jB} is given by,

$$\left[\frac{\partial\, v_{Ai}}{\partial\, v_{Bj}}\right]^{\Pi_{ABk}} = \beta_{ij}\,\frac{v_{Ai}}{v_{Bj}}\,.$$

In the *inter-regime-ensemble analysis* the reasoning is done between dimensionally linearly independent (DLI) variables from distinct ensembles, but within a fixed ensemble representation. In order to perform the inter-regime-ensemble reasoning, it is necessary **i)** to identify the *inter-ensemble-coupling-* or *inter-ensemble-contact-* variables and **ii)** the regimes in both ensembles that have the DLI variable forming a *chain* through the corresponding coupling regime. The inter-ensemble-coupling variables are exactly the variables that appear in the composition of the coupling regimes, i.e., the variables v_{Ai} and v_{Bj} above.

Once the DLI variables have been identified, the partials with respect to these variables can be computed as follows,

$$\frac{\partial\, v_{Ai}}{\partial\, v_{Bj}} = \frac{\partial\, v_{Ai}}{\partial\, v'_{Bk}}\,\frac{\partial\, v'_{Ak}}{\partial\, v'_{Bl}}\,\frac{\partial\, v'_{Bl}}{\partial\, v_{Bj}}\,,$$

[15]This analysis was called in [1] *inter-ensemble analysis.* We adopted a new terminology to fit with the extension introduced to across-ensemble analysis.

where v_{Ai}, v_{Bj} are DLI variables from distinct ensembles, and the prime $'$ denotes DLD variables. In fact, the expression above can be written in terms of the coefficients α's and β's. As we would need to consider 5 distinct cases leading to 5 distinct expressions (4 come from the consideration that the DLI variables are performance variables and 1 when they are basis variables), it would take much space and so, we have avoided to write them out. Nevertheless, they have been implemented in QDR.

There might be possible and sometimes of interest to reason with regimes (variables) that belong to the same ensemble but are in distinct representations. We call this analysis *intra-ensemble-representation* analysis. When the analysis is done with regimes from distinct ensembles and across their representations, we call this analysis *inter-ensemble-representation* analysis. Further discussions about these analyses will appear elsewhere, as presently they are not implemented in QDR.

Appendix B

In this appendix we give the interactive running session of QDR for the *Pressure Regulator* example. The output of QDR is comprehensible enough even for those readers that are not familiar with REDUCE.

The data file of a process requires the following three lists: `processlist`, `provarlist` and `dimreplist`.

The `processlist` is a list containing the names of the ensembles. The `provarlist` is the list containing the lists of variables of each ensemble. The `dimreplist` is a list containing the lists of dimensional representations of the corresponding ensemble variables.

To run QDR , one starts a fresh session of REDUCE, on the top of which one loads the system QDR and the input file. The system QDR has two modes: a *batch-like* mode and an *interactive* mode. The former mode is invoked within REDUCE with `qdr(nil);` and the latter with `qdr(t);`.

In the interactive mode the user must choose the performance variables and after that QDR starts running according to the algorithm presented in section 3 exhibiting step-by-step all calculations and analyses performed. For lack of space, we have left out the algebraic systems of homogeneity equations in the output given below.

In the batch-like mode, the user needs only to supply QDR with the input file and then it runs fully automatic all possible choices of performance variables presenting at the end a full account of the process (all regimes for all ensemble rerpresentations and respective analyses).

The input file for the pressure regulator, called `pregulat.dat`, is (the lines beginning with "%" are comments):

```
% Data file for the qualitative reasoning about the pressure regulator
% Ensemble A: pipe-orifice ensemble
% Ensemble B: spring-valve ensemble

processlist:={"Pressure regulator process. Pipe-orifice ensemble.",
              "Pressure regulator process. Spring-valve ensemble."}$
```

```
provarlist:={{pout,q,pin,aopen,rho},{x,pr,k}}$
dimreplist:={{mass*length**(-1)*time**(-2),length**3*time**(-1),
              mass*length**(-1)*time**(-2),length**2,mass*length**(-3)},
             {length,mass*length**(-1)*time**(-2),mass*time**(-2)}}$
;end;
```

Below we present the output of QDR.

```
   REDUCE 3.3  15-Jan-88

1: load "qdr"$

2: in "pregulator.dat"$

3: qdr(t);


      QDR - Qualitative Dimensional Reasoner

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               version of 07-Mar-91


Number of ensembles s=2.

Ensemble A
Process: Pressure regulator process. Pipe-orifice ensemble.

Process variables: {POUT,Q,PIN,AOPEN,RHO}
Number of process variables: n=5
Dimensional representations:
               -1          -2
[POUT] = LENGTH  *MASS*TIME
            3     -1
[Q] = LENGTH *TIME
              -1          -2
[PIN] = LENGTH  *MASS*TIME
                2
[AOPEN] = LENGTH
              -3
[RHO] = LENGTH  *MASS
Independent dimensions: {LENGTH,MASS,TIME}
Number of independent dimensions: d=3

Proceed to build the dimensional matrix? (Y OR N)
Y
Dimensional matrix:
      (POUT  Q PIN AOPEN RHO)
LENGTH(  -1  3  -1     2  -3)
MASS  (   1  0   1     0   1)
TIME  (  -2 -1  -2     0   0)
Rank of the dimensional matrix: r=3
Number of regimes: p=2
```

```
Proceed with the selection of performance variable(s)? (Y OR N)
Y
Select from the following list of process variables 2
performance variable(s):{POUT,Q,PIN,AOPEN,RHO}
The remaining 3 process variable(s) is(are) possible
candidate(s) to form the process' basis.

Do you want to select the process variable POUT as a performance
variable? (Y OR N)
Y

Do you want to select the process variable Q as a performance
variable? (Y OR N)
Y

The following process variable(s) was(were) chosen
as performance variable(s): {POUT,Q}, leading to {PIN,AOPEN,RHO}
as basis variables:

Proceed with Pi-calculus? (Y OR N)
Y
Pi-calculus:
           POUT
Pi(A,1) = ------
           PIN



             SQRT(RHO)*Q
Pi(A,2) = -----------------
           SQRT(PIN)*AOPEN

Proceed to build the extended dimensional matrix? (Y OR N)
Y
Extended dimensional matrix:

        (POUT  Q  PIN AOPEN RHO)
LENGTH  (  -1  3   -1     2  -3)
MASS    (   1  0    1     0   1)
TIME    (  -2 -1   -2     0   0)
Pi(A,1) (   1  0   -1     0   0)
Pi(A,2) (   0  1 -1/2    -1 1/2)

Proceed to analyse this matrix? (Y OR N)
Y
No variable was found to be superfluous.
The folowing variable(s) is(are) GLOBAL contact variable(s): {PIN}

Proceed with the intra-regime analysis? (Y OR N)
Y
Intra-regime analysis:
```

```
Regime Pi(A,1)

Partial of the performance variable POUT
with respect to the basis variable PIN:

            POUT
@POUT/@PIN=------.
            PIN

The performance variable POUT increases with power 1
as PIN increases.

Proceed? (Y OR N)
Y

Regime Pi(A,2)

Partial of the performance variable Q
with respect to the basis variable PIN:

           Q
@Q/@PIN=-------.
         2*PIN
                                                  1
The performance variable Q increases with power ---
                                                  2
as PIN increases.

Proceed? (Y OR N)
Y
Partial of the performance variable Q
with respect to the basis variable AOPEN:

             Q
@Q/@AOPEN=-------.
           AOPEN

The performance variable Q increases with power 1
as AOPEN increases.

Proceed? (Y OR N)
Y
Partial of the performance variable Q
with respect to the basis variable RHO:

              Q
@Q/@RHO= - -------.
            2*RHO
                                                  1
The performance variable Q decreases with power ---
                                                  2
as RHO increases.
```

```
Proceed with inter-regime analysis? (Y OR N)
Y
Inter-regime analysis:

Regimes Pi(A,1) and Pi(A,2) has(have) the following
contact variable(s): {PIN}.

Partial of the performance variable POUT
with respect to the performance variable Q
for contact variable PIN:

          2*POUT
@POUT/@Q=--------.
            Q

The performance variable POUT increases
as the performance variable Q increases
for contact variable PIN.

Proceed? (Y OR N)
Y

Do you want to try another representation? (Y OR N)
N

The following 1 representation(s) was(were) analysed
for this ensemble:
Performance variables   Basis variables
{POUT,Q}                {PIN,AOPEN,RHO}

Ensemble B
Process: Pressure regulator system. Spring-valve ensemble.

Process variables: {X,P,K}
Number of process variables: n=3
Dimensional representations:
[X] = LENGTH
            -1          -2
[P] = LENGTH  *MASS*TIME
               -2
[K] = MASS*TIME
Independent dimensions: {MASS,TIME,LENGTH}
Number of independent dimensions: d=3

Proceed to build the dimensional matrix? (Y OR N)
Y
Dimensional matrix:
      (X  P  K)
MASS  (0  1  1)
TIME  (0 -2 -2)
LENGTH(1 -1  0)
Rank of the dimensional matrix: r=2
Number of regimes: p=1
```

```
Proceed with the selection of performance variables? (Y OR N)
Y
Select from the following list of process variables 1
performance variable(s):{X,P,K}
The remaining 2 process variable(s) is(are) possible
candidate(s) to form the process' basis.

Do you want to select the process variable X as a performance
variable? (Y OR N)
Y

The following process variable(s) was(were) chosen
as performance variable(s): {X}, leading to {PR,K}
as basis variables:

Proceed with Pi-calculus? (Y OR N)
Y
Pi-calculus:
                  P*X
Regime Pi(B,1) = -----
                   K

Proceed to build the extended dimensional matrix? (Y OR N)
Y




Extended dimensional matrix:
        (X  P  K)
MASS    (0  1  1)
TIME    (0 -2 -2)
LENGTH  (1 -1  0)
Pi(B,1) (1  1 -1)

Proceed to analyse this matrix? (Y OR N)
Y
No variable was found to be superfluous.

Proceed with intra-regime analysis? (Y OR N)
Y
Intra-regime analysis:

Regime Pi(B,1)

Partial of the performance variable X
with respect to the basis variable P:

           X
@X/@PR= - ----.
           P
```

```
The performance variable X decreases with power 1
as P increases.

Proceed? (Y OR N)
Y
Partial of the performance variable X
with respect to the basis variable K:

       X
@X/@K=---.
       K

The performance variable X increases with power 1
as K increases.

Proceed with inter-regime analysis? (Y OR N)
Y
Inter-regime analysis is NOT possible with 1 regime!

Do you want to try another representation? (Y OR N)
N

The following 1 representation(s) was(were) analysed
for this ensemble:
Performance variables   Basis variables
{X}                     {P,K}

Proceed with intra-regime-ensemble analysis? (Y OR N)
Y
Intra-regime-ensemble analysis:
Ensembles 1 and 2 have the following coupling regimes:

                             POUT
Coupling regime Pi(AB,1) = ------
                              P

                             PIN
Coupling regime Pi(AB,2) = -----
                              P

                             AOPEN
Coupling regime Pi(AB,3) = -------
                               2
                              X

Partial of the process variable POUT
with respect to the process variable P:

           POUT
@POUT/@PR=------
            P
```

```
The process variable POUT increases
with power 1 as the process variable P increases.

Proceed? (Y OR N)
Y
Partial of the process variable PIN
with respect to the process variable P:

          PIN
@PIN/@PR=-----
           P

The process variable PIN increases
with power 1 as the process variable P increases.

Proceed? (Y OR N)
Y
Partial of the process variable AOPEN
with respect to the process variable X:

           2*AOPEN
@AOPEN/@X=---------
              X

The process variable AOPEN increases
with power 2 as the process variable X increases.

Proceed with inter-regime-ensemble analysis? (Y OR N)
Y
Inter-regime-ensemble analysis:

Ensembles A and B have the following inter-ensemble contact
variables: {POUT,P,PIN,AOPEN,X}.

Partial of the variable POUT with respect to X:

             POUT
@POUT/@X= - ------
              X

The variable POUT decreases as X increases.

Proceed? (Y OR N)
Y
Partial of the variable Q with respect to P:

        Q
@Q/@P=-----
       2*P

The variable Q increases as P increases.

Proceed? (Y OR N)
```

```
Y
Partial of the variable PIN with respect to X:

            PIN
@PIN/@X= - -----
             X

The variable PIN decreases as X increases.

Proceed? (Y OR N)
Y
Partial of the variable Q with respect to X:

       2*Q
@Q/@X=-----
        X

The variable Q increases as X increases.

Proceed? (Y OR N)
Y
Partial of the variable AOPEN with respect to P:

              2*AOPEN
@AOPEN/@P= - ---------
                 P

The variable AOPEN decreases as P increases.

Proceed? (Y OR N)
Y
Partial of the variable AOPEN with respect to K:

           2*AOPEN
@AOPEN/@K=---------
              K

The variable AOPEN increases as K increases.

Proceed? (Y OR N)
Y
Partial of the variable Q with respect to K:

       2*Q
@Q/@K=-----
        K

The variable Q increases as K increases.

The following 1 representation(s) was(were) analysed
for this process:

Ensemble A
```

```
Performance variables   Basis variables
{POUT,Q}                {PIN,AOPEN,RHO}

Ensemble B
Performance variables   Basis variables
{X}                     {P,K}

Proceed to build the process matrix? (Y OR N)
Y
Process matrix:
        (POUT  Q  PIN AOPEN RHO)( X  P  K)
LENGTH  (  -1  3   -1     2  -3   1 -1  0)
MASS    (   1  0    1     0   1   0  1  1)
TIME    (  -2 -1   -2     0   0   0 -2 -2)
Pi(A,1) (   1  0   -1     0   0   0  0  0)
Pi(A,2) (   0  1 -1/2    -1 1/2   0  0  0)
Pi(B,1) (   0  0    0     0   0   1  1 -1)
Pi(AB,1)(   1  0    0     0   0   0 -1  0)
Pi(AB,2)(   0  0    1     0   0   0 -1  0)
Pi(AB,3)(   0  0    0     1   0  -2  0  0)

Time: 50219 ms
```

The time above reflects the elapsed CPU time for a workstation Apollo DN3500, running Berkeley UNIX.

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